Metasurfaces and Wire-Media Metamaterials from Circuit Theory Perspective

Alexander B. Yakovlev
University of Mississippi
COLLABORATORS:

Mario G. Silveirinha
Stanislav I. Maslovski
George W. Hanson
Francisco Medina
Francisco Mesa
Andrea Alu
Igor S. Nefedov
Constantin R. Simovski
Sergei A. Tretyakov
Pavel A. Belov
Outline

• Introduction and Motivation
• Metasurfaces and Patterned Graphene for Enhanced Transmission
• Cloaking of Cylindrical Objects with Conformal Metasurfaces
• Homogenization Theory of Wire Media
• Applications of Wire-Media Metamaterials at Microwaves and THz Frequencies
• Conclusion
Homogenization of Structured Metasurfaces
Extremely thin conducting layers are almost opaque.


However, multilayer metal-dielectric PBG-like structures become transparent within certain frequency bands in the optical regime.

Microwave Transmissivity of a Metamaterial

Can a similar effect be observed at microwaves??


The metal films are substituted by perforated metal layers


Substrate thickness $(h)$: 6.35 mm
Period $(D)$: 5 mm
Strip width $(w)$: 0.15 mm
Dielectric permittivity: 3
Loss tangent = 0.0018
Thickness of grid = 18 µm

$h > D$
No higher-order mode coupling
**Circuit Model**

Typical waveguide problem with discontinuities

First higher order modes excited are TM\(_{02}\) and TE\(_{20}\)

\[ \lambda_c = D \]

\[ Z_0 \beta_0 \]

\[ Z_d \beta_d \]

\[ Z_g \]

\[ Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}; \quad Z_d = \frac{Z_0}{\sqrt{\varepsilon_r (1 - j \tan \delta)}} \]

\[ \beta_0 = \frac{\omega}{c}; \quad \beta_d = \beta_0 \sqrt{\varepsilon_r (1 - j \tan \delta)} \]

Effective Impedance

Dynamic model for effective impedance of the mesh grid

For $w \ll D$ and $D \ll \lambda$

Mesh grid behaves predominantly as an inductive load

Grid impedance

$$Z_g = R_g + j \omega L_g$$

$$R_g = \frac{D}{\left(\sigma w \delta_s\right)}$$

$$L_g = \frac{\eta_0 D}{2\pi c} \ln\left[\csc\left(\frac{\pi w}{2D}\right)\right]$$

$$\delta_s = \sqrt{\frac{2}{\omega \mu_0 \sigma}}$$

Skin depth

Power Transmission Spectra

What is the nature of these resonances??
- can we tune them?
- can we predict the band?
- The number of transmission peaks equals to the number of layers (resonators)

Bloch Waves

\[
cosh(\gamma d) = \cos(kd) + j \frac{Y_g Z_1}{2} \sin(kd)
\]

Upper Limit Condition

\[
cosh(\gamma d) = -1 \quad \Rightarrow \quad \cos(kd) = -1, \sin(kd) = 0
\]

\[
kd = \pi
\]

Lower Limit Condition

\[
\cos(kd) + j \frac{Y_g Z_1}{2} \sin(kd) = 1
\]
- Transmission band coincides with the finite structure
- Lower limit is influenced by grid impedance
- Upper limit is solely controlled by the slab thickness
Stacked Metallic Patches

Substrate thickness ($h$): 2 mm
Period ($D$): 2mm
Gap ($g$): 0.2 mm
Dielectric permittivity: 10.2
Loss tangent = 0.0035
Thickness of grid = 18 µm

Quasi-complementary version of the stacked mesh grid structure

$h >> g$  No higher order mode coupling

Typical waveguide problem with discontinuities

\[ Y_{0}^{\text{TE,TM}} = \cos \theta/\eta_{0}; \quad Y_{0}^{\text{TM}} = 1/(\eta_{0} \cos \theta) \]

\[ Y_{d}^{\text{TE,TM}} = \sqrt{\varepsilon - \sin^{2} \theta}/\eta_{0}; \quad Y_{d}^{\text{TM}} = \varepsilon_{r}/(\eta_{0} \sqrt{\varepsilon_{r} - \sin^{2} \theta}) \]
**Effective Impedance**

- **Dynamic model** for effective grid impedance of fish-net grid
- **Averaged impedance boundary condition**
- Approximate **Babinet** principle

\[\begin{align*}
D &= 2 \text{ mm}, \quad w = 0.2 \text{ mm}, \quad h = 1 \text{ mm} \\
\text{dielectric permittivity: 10.2}
\end{align*}\]

\[
C_g^{\text{TM}} = \frac{2D\varepsilon_{\text{eff}}}{\pi c\eta_0} \ln \left[\csc \left(\frac{\pi g}{2D}\right)\right]
\]

\[
C_g^{\text{TE}} = \frac{2D\varepsilon_{\text{eff}}}{\pi c\eta_0} \left(1 - \frac{\sin^2 \theta}{2\varepsilon_{\text{eff}}}\right) \ln \left[\csc \left(\frac{\pi g}{2D}\right)\right]
\]

\[R = \frac{D}{(D - g)\sigma \delta}\]

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WFA: Revisiting Equivalent Circuit Models for Emerging Technologies: From Microwaves to THz

IMS2014, Tampa, 1-6 June, 2014
In contrast with the band-pass behavior of the mesh grid structure, the patch structure exhibits a low-pass band followed by a wide stop-band.

Thickness ($h$): 2 mm  
Period ($D$): 2 mm  
Gap ($g$): 0.2 mm  
Dielectric permittivity: 10.2  
Loss tangent = 0.0035  
Thickness of grid = 18 µm

Electric Field Distributions
Graphene-Dielectric Stack

Atomically thin graphene sheet

Dielectric slab

Surface Conductivity of Graphene

Surface conductivity of graphene [Kubo formula]

$$\sigma(\omega, \mu_c, \Gamma, T) = \frac{j e^2 (\omega - j \Gamma)}{\pi \hbar^2} \left[ 1 - \frac{1}{(\omega - j \Gamma)^2} \int_{0}^{\infty} \left( \frac{\partial f_d(\varepsilon)}{\partial \varepsilon} - \frac{\partial f_d(-\varepsilon)}{\partial \varepsilon} \right) \varepsilon d\varepsilon \right]$$

Intraband contributions

$$\sigma_{\text{intra}} = -j \frac{e^2 k_B T}{\pi \hbar^2 (\omega - j \Gamma)} \left( \frac{\mu_c}{k_B T} + 2 \ln(e^{-\mu_c/k_B T} + 1) \right)$$

Interband contributions

$$\sigma_{\text{inter}} = -j e^2 \frac{2 |\mu_c|}{4 \pi \hbar} \ln \left( \frac{2 |\mu_c| - (\omega - j \Gamma) \hbar}{2 |\mu_c| + (\omega - j \Gamma) \hbar} \right)$$

- $e$: charge of electron, $T$: temperature, $\varepsilon$: energy
- $\omega$: angular frequency, $\hbar = \hbar/2\pi$: reduced Planck’s constant
- $\mu_c$: chemical potential, $\Gamma$: phenomenological scattering rate

In the far-infrared regime, the contribution due to the interband electron transition is negligible.

$$Z_s = 1/\sigma$$, at low-terahertz frequencies behaves as a low-loss inductive surface

G. W. Hanson, J. Appl. Phys., 103, 064302 (2008)
Surface Conductivity of Graphene

\[ \mu_c = 0.2 \text{ eV} \]

\[ \mu_c = 0.5 \text{ eV} \]

\[ \Gamma = \frac{1}{\tau} = 1.32 \text{ meV}, \quad \tau = 0.5 \text{ ps}, \quad T = 300 \text{ K} \]

\[ \sigma_{\text{min}} = \frac{\pi e^2}{2h} = 6.085 \times 10^{-5} \text{ S} \]

Solid lines: approximate closed-form expressions (intraband + interband)
Dashed lines: numerical integration [Kubo formula]
Free-Standing Graphene

Reflectivity and Transmissivity for normal incidence

Single sheet of graphene is highly reflective at low-THz frequencies
Behaves similar to an Inductive grid (metallic meshes) at microwaves

\[ \Gamma = \frac{1}{\tau} = 1.32 \, \text{meV} \]
\[ \tau = 0.5 \, \text{ps} \]
\[ T = 300 \, \text{K} \]
\[ \mu_c = 1 \, \text{eV} \]
Transmission resonance appears at low frequencies

FP-type resonance of dielectric slab loaded with graphene sheets

Graphene sheets effectively increase the electrical length

The number of transmission peaks is equal to the number of dielectric slabs within the characteristic frequency band.

- Thickness ($h$): 10 μm
- Permittivity: 10.2
- $\Gamma = 1/\tau = 1.32$ meV
- $\tau = 0.5$ ps, $T = 300$ K

$\mu_c = 1.0$ eV
Power Transmission Spectra

4 layer graphene structure
- 4 dielectric slabs
- 5 graphene sheets

Thickness ($h$): $10 \mu$m
Permittivity: $10.2$

\[ \Gamma = \frac{1}{\tau} = 1.32 \text{ meV} \]
\[ \tau = 0.5 \text{ ps}, \ T = 300 \text{ K} \]
Electric Field Distributions

\( \mu_c = 1.0 \text{ eV} \)
Electric Field Distributions

HFSS simulation results are obtained by Y. R. Padooru
Brillouin Diagram

**Multi-layer graphene-dielectric stack**

\[ \mu_c = 1.0 \text{ eV} \]

Thickness \((h)\): 10 \(\mu\)m
Permittivity: 10.2

SB: StopBand
PB: PassBand
Brillouin Diagram

Four-layer graphene-dielectric stack

$\mu_c = 1.0$ eV

SB: StopBand
PB: PassBand

A thick dielectric slab is needed for mechanical handling
Exhibits a series of bandpass regions separated by bandgaps

Thickness ($h$): 150 $\mu$m
Permittivity: 2.2
Five-layer graphene/meshgrid stacks separated by free-space

$h$: 30 μm, 
Period ($D$) = 20 μm, 
Strip width ($w$) = 2 μm, 
t = 0.4 μm, 
Dielectric permittivity: 1

✓ Graphene-air stack mimics the behavior of Fishnet-air stack at THz
Layered Graphene Patches

Metallic patches
- Capacitive surface reactance
- Microwave/low-terahertz frequencies

Metallic mesh-grids
- Inductive surface reactance
- Microwave/low-terahertz frequencies

Graphene sheets
- Low-loss inductive surface reactance
- Low-terahertz frequencies

Advantage of the Graphene Metasurface
Combining the properties of the above three structures in a single configuration

Combined filtering properties of the above three configurations
Surface Impedance of Graphene Patches

\[ Z_s = Z_{s1} + Z_{s2} \]
\[ = \frac{D}{(D - g)\sigma} - j \frac{\pi}{2\omega\varepsilon_0\varepsilon_r^{qs}} D \ln\left\{\csc\left(\frac{\pi g}{2D}\right)\right\} \]

Series R-L-C circuit

\[ Z_{s1} \left\{ \text{series R-L} \right\} \quad Z_{s2} \left\{ -\frac{j}{(\omega C_{eff})} \right\} \]

\[ C_{eff} = \frac{2}{\pi} \varepsilon_0 \varepsilon_r^{qs} D \ln\left\{\csc\left[\frac{\pi g}{(2D)}\right]\right\} \]

\[ \varepsilon_r^{qs} = \varepsilon_r \quad \text{For interior patches} \]

\[ \varepsilon_r^{qs} = (\varepsilon_r + 1)/2 \quad \text{For patches at top and bottom interfaces} \]
At low frequencies the behavior of graphene patches is similar to that of the metallic patches (capacitive).

At high frequencies the behavior of graphene patches becomes similar to that of a graphene sheet (inductive).
Transmissivity

\[ \mu_c = 0.5 \text{ eV} \]

\[ \mu_c = 1 \text{ eV} \]

Five-Layer Stack of Graphene Patches

\[ D = 10 \mu m; \, g = 1 \mu m; \, \varepsilon_r = 4; \, h = 10 \mu m \]

The analytical results are in good agreement with the full-wave simulations

Transmissivity

\[ \mu_c = 0.5 \text{ eV} \]

\[ \mu_c = 1 \text{ eV} \]

\[ D = 10 \mu m \]
\[ g = 1 \mu m \]
\[ \varepsilon_r = 4 \]
\[ h = 10 \mu m \]
Analytical Results

\[ D = 10 \mu m; \ g = 1 \mu m; \ \varepsilon_r = 4; \ h = 10 \mu m \]

- One can clearly notice the **low passband** (starting from zero frequency), followed by a deep **stopband**, and then a **second passband**
- Also, it can be noticed that with an increase in the **number of layers**, the number of **transmission peaks** which corresponds to the number of coupled layers **increases**, still maintaining the **same characteristic frequency bands**
Bloch-Wave Analysis

Analytical Results

\[ \cos(k_h h) = \cos(\theta) + j \frac{Z_d}{2Z_s} \sin(\theta) \]

The Brillouin diagrams perfectly predict the passband and stopband regions of the corresponding finite-layer structures

$D = 10 \, \mu m; \, g = 1 \, \mu m; \, \varepsilon_r = 4; \, h = 10 \, \mu m; \, \mu_c = 0.5 \, eV$

For mode A, the field value is zero near the middle graphene patch, which is consistent with the electric-field distribution shown in the inset.

For mode B, the field value is low in the middle graphene patch and is concentrated more near the remaining graphene patches, which is consistent with the electric-field distributions shown in the inset.
Surface Impedance and Transmissivity
Thin-Metal Patches

Four-layer thin-metal patch-dielectric stack

\[ D = 100 \text{ nm}; \ g = 10 \text{ nm}; \ \varepsilon_r = 1; \ h = 166 \text{ nm} \]

The results are calculated using a fit based on measured data [1] for the permittivity, utilizing an augmented Drude model [2]

Cloaking with a Metasurface

✓ The mantle cloaks are realized using various metasurfaces

Electric field distribution

Dielectric Cylinder: Conformal Slotted Metasurface Cloaks

\[ \begin{align*}
E & \quad H \\
\theta & \quad \phi \\
x & \quad y & \quad z
\end{align*} \]
Printed Planar Metasurfaces: Surface Impedance

Patch array

\[
Z_{s,\text{Patch}}^{TM} = \frac{\eta_0^2}{2 (\varepsilon_r + 1)} Z_{s,\text{Mesh}}^{TE}
\]

\[
Z_{s,\text{Patch}}^{TE} = \frac{\eta_0^2}{2 (\varepsilon_r + 1)} Z_{s,\text{Mesh}}^{TM}
\]

Mesh Grid

\[
Z_{g, TE}^{TM} Z_{g, TM}^{TE} = \frac{\eta_{\text{eff}}^2}{4}
\]

Babinet's Principle

Jerusalem crosses

\[
Z_{s,\text{JC}}^{TM} = j \omega L_{g, TM}^{TM} + \frac{1}{j \omega C_{g}}
\]

\[
Z_{s,\text{JC}}^{TE} = j \omega L_{g, TE}^{TE} + \frac{1}{j \omega C_{g}}
\]

\[
L_{g, TM}^{TE} = \frac{\eta_0 D}{2c\pi} \ln \csc \left( \frac{\pi w}{2D} \right) \left( 1 - 2 \frac{\sin^2 \theta_s}{\varepsilon_r + 1} \right)
\]

\[
L_{g, TM}^{TM} = \frac{\eta_0 D}{2c\pi} \ln \csc \left( \frac{\pi w}{2D} \right)
\]

\[
C_{g} = \frac{\varepsilon_0 \varepsilon_r d}{\pi} \left[ \ln \csc \left( \frac{\pi g}{2D} \right) + F \right]
\]

\[
F = \frac{\sqrt{1 - \left( \frac{d}{\lambda} \right)^2} u^2}{1 + \sqrt{1 - \left( \frac{d}{\lambda} \right)^2} (1 - u)^2} + \left[ \frac{d u (3 u - 2)}{4 \lambda} \right]^2
\]

\[
u = \cos^2 \left( \frac{\pi g}{2d} \right)
\]

\[
\lambda = k_0 \sqrt{\frac{\varepsilon_r + 1}{2}}
\]

Printed Planar Metasurfaces: Surface Impedance

Cross dipoles

\[
\begin{align*}
Z_{s,\text{TM,Dipole}} &= j\omega L_{TM} + \frac{1}{j\omega C}, \\
Z_{s,\text{TE,Dipole}} &= j\omega L_{TE} + \frac{1}{j\omega C} \\
C_g &= \frac{1}{\left(\frac{1}{C} - \frac{0.3595l_{\text{eff}}^2}{\varepsilon_0 (\varepsilon_r + 1) D^3}\right)} \\
C &= \frac{\pi\varepsilon_0\varepsilon_{\text{eff}}}{4 \ln \left(\frac{4l}{w}\right)}
\end{align*}
\]

Babinet’s Principle

\[
Z_{s,\text{TE,Slot Dipole}} = \frac{\eta_0^2}{2 (\varepsilon_r + 1) Z_{s,\text{TE,Dipole}}} \\
Z_{s,\text{TE,Slot JC}} = \frac{\eta_0^2}{2 (\varepsilon_r + 1) Z_{s,\text{TM,JC}}} \\
Z_{s,\text{TM,Slot JC}} = \frac{\eta_0^2}{2 (\varepsilon_r + 1) Z_{s,\text{TM,JC}}}
\]


Slot Jerusalem crosses

Slot cross dipoles
WFA: Revisiting Equivalent Circuit Models for Emerging Technologies: From Microwaves to THz

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Boundary Conditions:

- continuity of tangential components of electric and magnetic fields at the boundary of the core ($\rho = a$)
- *two-sided impedance boundary conditions* at the interface of the mantle cloak ($\rho = a_c$)

Two-sided impedance boundary condition:

$$E_t|_{\rho=a_c^+} = E_t|_{\rho=a_c^-} = Z_s \left( H_t|_{\rho=a_c^+} - H_t|_{\rho=a_c^-} \right)$$

$Z_s$ is the surface impedance of the FSS mantle cloaks

Scattering Width

$$\sigma_s = \frac{4}{k_0} \left| \sum_{n=-\infty}^{\infty} c_n^{TM} \right|^2$$

By adjusting the values of $a_c$, $\varepsilon_c$, and $X_s$

$$Z_s = R_s + jX_s$$

For lossless case ($R_s = 0$):

$$Z_s = jX_s$$
Dielectric Cylinder: Conformal Slotted Metasurface Cloaks

- TM Slot JC
- TE Slot JC
- TM Slot cross dipole
- TE Slot cross dipole

Solid lines: Analytical model
Dashed lines: HFSS
Electric Field Distributions

HFSS simulation results are obtained by Y. R. Padooru

Conducting Cylinder: Conformal Printed Metasurface Cloaks

Patch array

Jerusalem crosses

Cross dipoles

PEC
Electric Field Distributions

HFSS Simulation Results

Cloak | TM polarization | No cloak

HFSS simulation results are obtained by Y. R. Padooru

Cloaking of Cylinders: Graphene Nanostructured Metasurface
Electric Field Distributions

Graphene Nanostructured Metasurface

Cloak

No Cloak

Metallic Cylinder

CST simulation results are obtained by Pai-Yen Chen

Electric Field Distributions

Graphene Nanostructured Metasurface

Dielectric Cylinder

Cloak

No Cloak

CST simulation results are obtained by Pai-Yen Chen
Wire Media Based Metamaterials


What is “Wire Media”? 

Structured materials formed by metallic rods 

Uniaxial WM formed by parallel wires 

\[ a \ll \lambda \]
Homogenization of Wire Media

For long wavelengths the propagation of waves can be described using an effective medium approach

\[ \varepsilon \]

Tensor effective permittivity
Uniaxial WM: Local and Nonlocal Models

Local dielectric function:
\[
\frac{\overline{\varepsilon}}{\varepsilon_0} = \varepsilon_h \left( \bar{I}_t + \left(1 - \frac{k_p^2}{k_h^2}\right) \hat{z} \hat{z} \right)
\]

Nonlocal dielectric function:
\[
\frac{\overline{\varepsilon}}{\varepsilon_0} = \varepsilon_h \left( \bar{I}_t + \left(1 - \frac{k_p^2}{k_h^2 - k_z^2}\right) \hat{z} \hat{z} \right)
\]

**Plasma wavenumber** depends on the geometrical parameters of the wire lattice
\[
k_p = \sqrt{\frac{(2\pi/a)^2}{\ln(a/2\pi r_0) + 0.5275}}
\]

**Problem Involving Interfaces**

- Dielectric function is defined only for an unbounded periodic wire medium
- In general, a spatially dispersive material may support “new waves”

Wire medium supports three types of modes

- **TE-z (ordinary mode)**
  The *ordinary mode* does not interact with the wires, and ‘sees’ an effective medium as the host medium.

  \[
  k_{z,TE} = \sqrt{(\omega/c)^2 \varepsilon_h^2 - k_x^2 - k_y^2}
  \]

- **TM-z (extraordinary mode)**
  The *extraordinary mode* interacts with the wires, and corresponds to nonzero currents in the wires and nonzero electric field along the wires.

  \[
  k_{z,TM} = -j \sqrt{k_p^2 + k_x^2 + k_y^2 - (\omega/c)^2 \varepsilon_h^2}
  \]

- **TEM (transmission line mode)**
  Corresponds to nonzero currents in the wires and zero electric field along the wires, and it is defined for any wave vector in the transverse direction.

  \[
  k_{z,TEM} = (\omega/c)\sqrt{\varepsilon_h}
  \]
Quasi-Static Modeling of WM

- Full electromagnetic description of the WM and the transmission line analogy
- Formulated in terms of effective capacitance $C$ and effective inductance $L$ per unit length

\[
\int E \cdot dl = \int_{z}^{z+\Delta z} E_z(r_0, z') dz' - \int_{z}^{z+\Delta z} E_z(a/2, z') dz' + \int_{r_0}^{a/2} E_x(x, z+\Delta z) dx - \int_{r_0}^{a/2} E_x(x, z) dx
\]

\[
\int E \cdot dl = Z_w \Delta z \langle I_z \rangle - E_z \Delta z + \varphi(z + \Delta z) - \varphi(z)
\]

\[
\frac{\partial \langle \varphi \rangle}{\partial z} = -(j \omega L + Z_w) \langle I_z \rangle + E_z
\]

\[
\int E \cdot dl = -j \omega \varphi = -j \omega LI_z \Delta z
\]

S. I. Maslovski and M. G. Silveirinha, Phys. Rev. B, 80, 245101, 2009
Local Framework for Nonlocal WM

Field equations in a uniaxial WM with wires oriented along the $z$-axis

$$\nabla \times E = -j\omega \mu_0 H$$

$$\nabla \times H = j\omega \varepsilon_0 E + J$$

$$J = \frac{\langle I_z \rangle}{A_{cell}} \hat{z}$$

$$\frac{\partial \langle \phi \rangle}{\partial z} = -(j\omega L + Z_w)\langle I_z \rangle + E_z$$

$$\frac{\partial \langle I_z \rangle}{\partial z} = -j\omega \langle q \rangle = -j\omega C \langle \phi \rangle$$

$$L = (\mu_0/2\pi) \log \left[ \frac{a^2}{4r_0} (a - r_0) \right]$$  Effective wire Inductance per unit length

$$C = 2\pi \varepsilon_0 / \log \left[ \frac{a^2}{4r_0} (a - r_0) \right]$$  Effective wire Capacitance per unit length

✓ Applicable to wire media with attached conducting bodies
✓ Loading is assumed to be effectively continuous along the wires

S. I. Maslovski and M. G. Silveirinha, Phys. Rev. B, 80, 245101, 2009
Nonlocal Framework for Nonlocal WM

Permittivity dyadic

We express the $z$-component of the electric displacement as

$$D_z = \varepsilon_0 E_z + \frac{J_z}{j\omega} \quad J_z = \langle I_z \rangle / A_{cell}$$

$$E_z = (j\omega L + Z_w)\langle I_z \rangle + \frac{\partial \langle \varphi \rangle}{\partial z} \quad \frac{\partial \langle I_z \rangle}{\partial z} = -j\omega C\langle \varphi \rangle$$

The in-plane component of the electric displacement

$$D_t = \varepsilon_0 E_t$$

Finally

$$\varepsilon = \langle I_t \rangle + \left(1 - \frac{k_p^2}{k_0^2 - j\xi k_0 - k_z^2 / n^2}\right) z_0 z_0$$

$$k_p^2 = \mu_0 / A_{cell} L, \quad \xi = Z_w \sqrt{\varepsilon_0 \mu_0} / L, \quad n^2 = LC / \varepsilon_0 \mu_0$$

Effective wire Inductance per unit length

$$L = (\mu_0 / 2\pi) \log \left[ a^2 / 4r_0 (a - r_0) \right]$$

Effective wire Capacitance per unit length

$$C = 2\pi \varepsilon_0 / \log \left[ a^2 / 4r_0 (a - r_0) \right]$$

S. I. Maslovski and M. G. Silveirinha, Phys. Rev. B, 80, 245101, 2009
Additional Boundary Conditions

The scattering problem is undetermined!

\[ \varepsilon_{zz}(\omega, k_z) = 1 - \frac{k_p^2}{(\omega/c)^2 - k_z^2} \]

2 distinct polarizations

3 distinct polarizations
Additional Boundary Conditions (ABCs)

Double-Sided Wire Junction at Wire-to-Patch Interface

\[ \frac{dI_1(z)}{dz} \bigg|_{z_0^-} + \frac{dI_2(z)}{dz} \bigg|_{z_0^+} = \frac{2C}{C_{patch}} \left[ I_2(z_0^+) - I_1(z_0^-) \right] = 0 \]

\[ \frac{dI_2(z)}{dz} \bigg|_{z_0^+} - \frac{dI_1(z)}{dz} \bigg|_{z_0^-} = 0 \]

\[ C = 2\pi\varepsilon_h \varepsilon_0 / \log \left[ \frac{a^2}{4r_0} (a - r_0) \right] \]

\[ C_{patch} = 2\pi\varepsilon_0 \varepsilon_h (a - g) / \log \left[ \sec \left( \pi g / 2a \right) \right] \]

Effective wire Capacitance per unit length

Capacitance of the patch

S. I. Maslovski et al., New J. Phys., 12, 113047, 2010
Double-Sided Wire Junction with Lumped Impedance Insertions

\[ \frac{dI_2(z_0^+)}{dz} - \frac{dI_1(z_0^-)}{dz} = j\omega C \left[ I_1(z_0^-)Z_1 + I_2(z_0^+)Z_2 \right] \]

\[ \frac{dI_1(z_0^-)}{dz} + \frac{dI_2(z_0^+)}{dz} = \frac{2C}{C_{\text{patch}}} \left[ I_2(z_0^+) - I_1(z_0^+) \right] - j\omega C \left[ I_2(z_0^+)Z_2 - I_1(z_0^-)Z_1 \right] \]

- \( C = 2\pi\varepsilon_h\varepsilon_0 \left/ \log \left[ a^2 / 4r_0 (a - r_0) \right] \right. \)
- \( C_{\text{patch}} = 2\pi\varepsilon_0 \varepsilon_h (a - g) / \log \left[ \sec (\pi g / 2a) \right] \)

Effective wire Capacitance per unit length

Capacitance of the patch
Additional Boundary Conditions (ABCs)

Double-Sided Wire Junction at Wire-to-Resistive-Sheet Interface

Thin resistive sheet

medium 1

wire 1

medium 2

wire 2

Thin metal

Leontovitch Boundary Condition:

\[ \vec{E}_t = \frac{1}{\sigma_{2D}} \vec{J}_s \quad \sigma_{2D} = \sigma_{3D} t \]

Principle of Conservation of Surface Charge

\[ \rho_{s1,2} = -\frac{\nabla_s \cdot \vec{J}_{w1,2}}{j\omega} \]

Kirchoff’s Current Law

\[ J_s = J_{w1} - J_{w2} \]


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Negative Refraction with Mushroom Structure

\[ a = 2 \text{ mm, } \varepsilon_h = 10.2, \ g = 0.1a, \ r = 0.025a, \ \omega a/c = 0.45 \]


Gaussian Beam Simulations

\[ a = 2 \text{ mm}, \quad \varepsilon_h = 10.2, \quad g = 0.1a, \quad r = 0.025a, \quad \omega a/c = 0.45, \quad \theta_1 = 32^\circ \]

CST simulation results are obtained by Mario G. Silveirinha
Mushroom Structure with Loaded Vias

**Loads (Inductive)**
- Increase the inductance of wires
- Significantly reduce the SD effects

\[
a/h > 1 \\
k_0 \sqrt{\varepsilon_h} h \ll \pi
\]

\[f = 11 \text{GHz}\]

Lumped loads

\[\theta_i = 33^\circ\]
\[\theta_t = -65.4^\circ\]

Gaussian Beam Simulations

\[ a = 2 \text{ mm}, \; g = 0.2 \text{ mm}, \; h = 2 \text{ mm}, \; r_0 = 0.05 \text{ mm}, \; \epsilon_h = 1, \; \text{Load} = 5\text{nH} \; \text{and} \; f = 11 \text{ GHz} \]

Width of the array along \( x, \; W_x = 90 \; a \)

CST simulation results are obtained by Mario G. Silveirinha
Reflection Phase Characteristics

The load is connected to the ground plane through a gap of 0.1 mm, by curve fitting, the correction terms are $L_{\text{par}} = 0.02$ nH and $C_{\text{par}} = 0.06$ pF.

- Reflection phase depend strongly on the value and the type of load.
- For an increase in the value of the inductive loads, we have a decrease in the plasma frequency and reduction in SD effects.

C. S. R. Kaipa et. al., IEEE AWPL, 10, 1503, 2011

Thickness ($h$) = 1 mm
Period ($a$) = 2 mm
Gap ($g$) = 0.2 mm,
Permittivity $\epsilon_h = 10.2$
Reflection Characteristics

- The resonance frequency is shifted to a much lower value, when compared to the structure without loads.
- The effects of the parasitic are negligible.
- Electrical thickness of the structure at the operating frequencies for 5 nH load is $\lambda_0/30$

WFA: Revisiting Equivalent Circuit Models for Emerging Technologies: From Microwaves to THz

C. S. R. Kaipa et. al., IEEE AWPL, 10, 1503, 2011

Thickness ($h$) = 1 mm
Period ($a$) = 2 mm
Gap ($g$) = 0.2 mm,
Permittivity $\epsilon_h = 1$
Partial Focusing Lens

7-layer structure
8-patch arrays
7-Inductive loaded WM slabs

\[ a = 2 \text{ mm}, \quad g = 0.2 \text{ mm}, \quad h = 2 \text{ mm}, \quad r_0 = 0.05 \text{ mm} \]

\[ \epsilon_h = 1, \quad \theta_i = 45 \, \text{deg}, \quad \text{and Load} = 5nH \]

Transmission Characteristics

7-layer structure
8-patch arrays
7-Inductive loaded WM slabs

- Exhibits high transmission
- Frequency of operation is chosen such that transmission angle is reasonably a linear function of incidence angle

\( a = 2 \text{ mm}, g = 0.2 \text{ mm}, h = 2 \text{ mm}, r_0 = 0.05 \text{ mm}, \epsilon_h = 1, \theta_i = 45 \text{ deg}, \text{ and Load} = 5nH \)
Magnetic-Field Profile

7-layer structure \( f = 10 \text{ GHz}, d = 0.23\lambda_0, L = 0.48\lambda_0 \)

\( a = 2 \text{ mm}, g = 0.2 \text{ mm}, h = 2 \text{ mm}, r_0 = 0.05 \text{ mm}, \epsilon_h = 1, \) and Load = 5nH

HFSS simulation results are obtained by C. S. R. Kaipa
Amplification of Evanescent Waves

- Strong amplification of the near field for the wave vector components in the range $1 < k_x/k_0 < 4$ at the frequency of 6.67 GHz


Graphs showing the amplification at 6.67 GHz and 5.8 GHz with parameters: $a = 2$ mm, $g = 0.2$ mm, $r_0 = 0.05$ mm, $L1 = 5$ nH, $L = 10$ mm, $\epsilon_h = 1$
Subwavelength Imaging with Inductive Loadings

Solid lines: Analytical model
Dashed lines: HFSS

Resolution of the bi-layer mushroom lens is $\lambda_0/6$
Subwavelength Imaging with Inductive Loadings

Width of the slab along $x$, $W_x = 1.8\lambda_0$

$f = 6.67$ GHz, Inductive loading of 5 nH

$\lambda = \frac{d}{\sqrt{L}}$

$L = 0.222\lambda_0$

$d = 0.11\lambda_0$

HFSS simulation results are obtained by C. S. R. Kaipa
Wideband Absorbers

Two-layer Mushroom HIS absorber

Three-layer Mushroom HIS absorber

Conclusions

✓ Printed and slotted metasurfaces and graphene patterned surfaces are analyzed with simple circuit models demonstrating the enhanced transmission at microwave and terahertz frequencies, and can be effectively used for cloaking of dielectric and conducting objects.

✓ We review recent contributions to the homogenization theory of wire media and discuss a variety of applications at microwave and THz frequencies.

✓ The applications include, but are not limited to, metamaterial-based antenna and waveguide technology, sub-wavelength imaging, super lenses, absorbers, and have been recently extended to carbon-based nanomaterials.