Metasurface Cloaks for Dielectric and Metallic Elliptical Cylinders and Strips

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Outline

- Introduction and Motivation
  - Mantle Cloaking

- Formulation and Theory
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  - Formulation of the Scattering Problem
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- Analytical and Full-wave Simulation Results
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  - Strip as a Degenerated Ellipse

- Conclusions
Cloaking Methods I

- **Transformation Optics**
  - Manipulation of electromagnetic energy flow
  - Distorting the ray path by using bulk metamaterials


- **Plasmonic Method**
  - Based on scattering cancellation
  - Homogeneous and isotropic materials
  - Low or negative index materials
  - Suppression of the dominant mode


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Cloaking Methods II

- Mantle Cloaking
  - Based on scattering cancellation
  - An ultrathin metasurface
  - Anti-phase surface currents
  - Suppression of the dominant mode


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1-D and 2-D Periodic Structures

Vertical Strips

Mesh Grids

Capacitive Rings

Patch Arrays


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Mantle Cloaking using Graphene

- The thinnest possible mantle cloak
- Large tunability
- Applicable for low terahertz (THz) frequencies


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Graphene Nanopatches

- Graphene monolayer provides inductive surface impedance
- A conducting object needs capacitive surface impedance to be cloaked
- To resolve this issue, a patterned graphene metasurface is proposed, which owns dual capacitive/inductive inductance and can be used to cloak both dielectric and conducting objects

\[ Z_s = R_s - jX_s \]
\[ Z_s = \frac{D}{\sigma_s(D-g)} + j \frac{\pi}{2\omega\varepsilon_0(\varepsilon_r+1)}D \ln[csc\left(\frac{\pi g}{2D}\right)] \]

- \( R_s \): surface resistance per unit cell
- \( X_s \): surface reactance per unit cell
- \( D \): periodicity size
- \( g \): gap size
- \( \varepsilon_r \): relative permittivity of the dielectric cylinder or the spacer

Cloaking using Graphene Nanopatches


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Elliptical Cloak Designs at Microwaves

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Domain-Product Technique Solution for the Problem of Electromagnetic Scattering From Multiangular Composite Cylinders

Vitaliy P. Chumachenko, Member, IEEE

Abstract—This paper presents a domain-product technique solution for the problem of electromagnetic scattering from a two-dimensional structure composed of multiple perfect conductors and lossless dielectrics with arbitrarily polyhedral boundaries. The configuration is assumed to be excited by a plane wave polarized transverse magnetic or transverse electric to the axis of the cylinders. For both the interior and exterior subregions, efficient field representations are attained in the forms of Mathieu function expansions. A system of infinite matrix equations with respect to the expansion coefficients results from the boundary conditions. Solution to the system is found using a truncation procedure. Numerical examples are presented that demonstrate the validity, flexibility, and capability of the technique. In the middle frequency range, the approach proposed enables accurate numerical analysis of fairly complicated structures with low computational cost.

Index Terms—Cylinders, domain-product technique, electromagnetic theory.

Fig. 1. Geometry of composite cylindrical scatterer and pertinent coordinate systems.
Elliptical Coordinate System

\[ F = \sqrt{a_0^2 - b_0^2} \]

- \( x = F \cosh(u) \cos(v) \)
- \( y = F \sinh(u) \sin(v) \)
- \( u_0 = \tanh^{-1}(b_0/a_0) \)
- \( h = F \sqrt{\cosh(u)^2 - \cos(v)^2} \)
- \( h = h_u = h_v \)
- \( h_z = 1 \)

Scale factors

- \( u = u_0 \) : represents an ellipse
- \( v = v_0 \) : represents a hyperbola

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Mathieu Equation

- Two-dimensional Helmholtz Equation:
  \[(\nabla^2 + K^2)E = 0\]
- where:
  \[\nabla^2 = \frac{1}{h^2} \left( \frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2} \right)\]
- Using the method of separation of variables we have:
  \[E(u, v) = R(u) S(v)\]

  \[q = \frac{K^2 F^2}{4}\]

  \[R'' - R(A - 2q \cosh(2u)) = 0\]
  \[S'' + S(A - 2q \cos(2u)) = 0\]

- The radial Mathieu equation has four kinds of solution as:
  \[J_{pm}(q, u, n) \quad Y_{pm}(q, u, n) \quad H^{(1)}_{pm}(q, u, n) \quad H^{(2)}_{pm}(q, u, n)\]
- The angular Mathieu equation has the solution as:
  \[S_{pm}(q, \nu, n)\]

\[p, m \text{ can be even or odd}\]
Formulation of the Scattering Problem

\[ E_z^i = \sqrt{8\pi} \sum_n j^{-n} \frac{J_{pm}(q_0, u, n)}{N_{pm}(q_0, n)} S_{pm}(q_0, v, n) S_{pm}(q_0, \varphi, n); \quad u > u_0 \]

\[ E_z^s = \sqrt{8\pi} \sum_n j^{-n} a_{pm} H_{pm}^{(1)}(q_0, u, n) S_{pm}(q_0, v, n) S_{pm}(q_0, \varphi, n); \quad u > u_0 \]

\[ H_V = \frac{1}{j \omega \mu} \left( -\frac{\partial}{\partial u} E_z \right) = -\frac{1}{j \omega \mu} \frac{\partial}{\partial u} E_z = \frac{1}{j \omega \mu} \frac{\partial}{\partial u} E_z \]

\[ H_V^i = \frac{j}{\omega \mu} \sqrt{8\pi} \sum_n j^{-n} \frac{J'_{pm}(q_0, u, n)}{N_{pm}(q_0, n)} S_{pm}(q_0, v, n) S_{pm}(q_0, \varphi, n) \]

\[ H_V^s = \frac{j}{\omega \mu} \sqrt{8\pi} \sum_n j^{-n} a_{pm} H_{pm}^{(1)}(q_0, u, n) S_{pm}(q_0, v, n) S_{pm}(q_0, \varphi, n) \]

\[ E_z^t = \sqrt{8\pi} \sum_n j^{-n} [b_{pm} J_{pm}(q_1, u, n) + c_{pm} Y_{pm}(q_1, u, n)] S_{pm}(q_1, v, n) S_{pm}(q_0, \varphi, n); \quad u_1 < u < u_0 \]

\[ H_V^t = \frac{j}{\omega \mu} \sqrt{8\pi} \sum_n j^{-n} [b_{pm} J'_{pm}(q_1, u, n) + c_{pm} Y'_{pm}(q_1, u, n)] S_{pm}(q_1, v, n) S_{pm}(q_0, \varphi, n) \]

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Boundary Conditions

- By applying boundary conditions:

\[ E_{t_{lu=u_1}} = 0 \rightarrow b_{pm} J_{pm} (q_1, u_1, n) + c_{pm} Y_{pm} (q_1, u_1, n) = 0 \]

\[ E^i + E^s_{lu=u_0} = E^t_{lu=u_0} \]

\[ \sqrt{8\pi} \sum_n j^{-n} \left[ \frac{J_{pm} (q_0, u_0, n)}{N_{pm} (q_0, n)} + a_{pm} H_{pm}^{(1)} (q_0, u_0, n) \right] S_{pm} (q_0, v, n) S_{pm} (q_0, \varphi, n) \]

\[ = \sqrt{8\pi} \sum_n j^{-n} \left[ b_{pm} J_{pm} (q_1, u_0, n) + c_{pm} Y_{pm} (q_1, u_0, n) \right] S_{pm} (q_1, v, n) S_{pm} (q_0, \varphi, n) \]

\[ Z_s \left[ H^i + H^s_{lu=u_0} - H^t_{lu=u_0} \right] = E^t_{lu=u_0} \rightarrow \text{Sheet Impedance Boundary Condition} \]

\[ Z_s \left\{ \frac{j\sqrt{8\pi}}{\omega \mu h_0} \sum_n j^{-n} \left[ J'_{pm} (q_0, u_0, n) \right] S_{pm} (q_0, v, n) S_{pm} (q_0, \varphi, n) - \right. \]

\[ \left. \frac{j\sqrt{8\pi}}{\omega \mu h_0} \sum_n j^{-n} \left[ b_{pm} J'_{pm} (q_1, u_0, n) + c_{pm} Y'_{pm} (q_1, u_0, n) \right] S_{pm} (q_1, v, n) S_{pm} (q_0, \varphi, n) \right\} = \]

\[ \sqrt{8\pi} \sum_n j^{-n} \left[ b_{pm} J_{pm} (q_1, u_0, n) + c_{pm} Y_{pm} (q_1, u_0, n) \right] S_{pm} (q_1, v, n) S_{pm} (q_0, \varphi, n) \]
Now, we apply the orthogonality property of angular Mathieu function as below:

\[ \int_0^{2\pi} S_{p_m'}(q_0, v, n) S_{p_m}(q_1, v, n) = \gamma_{p_m} \delta_{mm'}, \delta_{mm'} = \begin{cases} 0; & m \neq m' \\ 1; & m = m' \end{cases} \]

After some manipulation we come to the matrix equation below:

\[
\begin{bmatrix}
0 \\
J_{p_m}(q_1, u_1, n) \\
-\frac{\gamma_{p_m}}{N_{p_m}} J_{p_m}(q_1, u_0, n) \\
H_{p_m}^{(1)}(q_0, u_0, n) \\
-\frac{\gamma_{p_m}}{N_{p_m}} (J'_{p_m}(q_1, u_0, n)) - \frac{\gamma_{p_m}}{N_{p_m}} \left( \frac{\omega \mu h_0}{j Z_s} J_{p_m}(q_1, u_0, n) \right) \\
-\frac{\gamma_{p_m}}{N_{p_m}} (J'_{p_m}(q_1, u_0, n)) - \frac{\gamma_{p_m}}{N_{p_m}} \left( \frac{\omega \mu h_0}{j Z_s} Y_{p_m}(q, u_0, n) \right) \\
\end{bmatrix} \begin{bmatrix}
A_{p_m} \\
B_{p_m} \\
C_{p_m} \\
\end{bmatrix}
\]

Then, unknown coefficients are found
Bistatic Scattering Width

- To find the scattered field for farfield region, we use the asymptotic form of the radial Mathieu function and we have:

\[
\overrightarrow{E^s} = \frac{\sqrt{8\pi}}{\sqrt{K\rho}} e^{j(K\rho - \frac{\pi}{4})} \sum_n j^{-2n} a_{pm} S_{pm}(q_0, \nu, \eta) S_{pm}(q_0, \varphi, \eta) \hat{\mathbf{\hat{z}}}
\]

It has already been found by solving the matrix equation

- The two-dimensional bistatic cross section is defined as:

\[
\sigma_{2D} = \lim_{\rho \to \infty} 2\pi \rho \frac{|E^s|^2}{|E^i|^2}
\]

- Finally, we have:

\[
\frac{\sigma_{2D}}{\lambda} = \left[ \sum_n \sqrt{8\pi} j^{-2n} a_{pm} S_{pm}(q_0, \nu, \eta) S_{pm}(q_0, \varphi, \eta) \right]^2
\]
Optimum Reactance in terms of Observation Angle

- To find the optimum reactance versus observation angle, we solve the matrix equation for the dominant scattering mode and require

$$a_{ee}(n = 0) = 0$$

- Finally, the optimum required reactance versus observation angle can be found as:

$$Z_s = j \omega \mu \sqrt{\cosh^2 u_0 - \cos^2 \nu \times F} \times \frac{(Y_{pm}(q_1, u_1, n)Y_{pm}(q_1, u_0, n) - Y_{pm}(q_1, u_1, n)J_{pm}(q_1, u_0, n))}{J_{pm}(q_1, u_1, n)\left(Y'_{pm}(q_1, u_0, n) - \frac{Y_{pm}(q_1, u_0, n)J'_{pm}(q_0, u_0, n)}{J_{pm}(q_0, u_0, n)}\right) + Y_{pm}(q_1, u_1, n)\left(-J'_{pm}(q_1, u_0, n) + \frac{J_{pm}(q_1, u_0, n)J'_{pm}(q_0, u_0, n)}{J_{pm}(q_0, u_0, n)}\right)}$$

- With the same approach, we can find the optimum required reactance versus observation angle for the dielectric elliptical cylinder as:

$$Z_s = \omega \mu h_0 \frac{J_{pm}(q_0, u_0, n)J_{pm}(q_1, u_0, n)}{J_{pm}(q_0, u_0, n)J'_{pm}(q_1, u_0, n) - J_{pm}(q_1, u_0, n)J'_{pm}(q_0, u_0, n)}$$

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To find for what value of $Z_s$, the scattering cancelation occurs for all observation angles, we should find where $E_Z^s$ becomes minimum:

$$E_Z^s = \sqrt{8\pi j^{-n}} a_{pm}(Z_s(v)) H_{pm}^{(1)}(q_0, u, n) S_{pm}(q_0, v, n) S_{pm}(q_0, \varphi, n); \; u > u_0$$

$$\frac{d a_{ee}(n = 0)}{d v} = 0 \rightarrow \frac{d Z_s}{d v} = 0 \rightarrow$$

**PEC**

$$Z_{s-opt} = j \omega \mu F \cosh u_0 \times$$

$$\frac{J_{pm}(q_1, u_1, n) Y_{pm}(q_1, u_1, n) - Y_{pm}(q_1, u_1, n) J_{pm}(q_1, u_1, n)}{J_{pm}(q_0, u_0, n)} + Y_{pm}(q_1, u_1, n) \left( -J'_{pm}(q_1, u_1, n) + \frac{J_{pm}(q_1, u_0, n) J'_{pm}(q_0, u_0, n)}{J_{pm}(q_0, u_0, n)} \right)$$

**Dielectric**

$$Z_{s-opt} = j \omega \mu F \cosh u_0 \frac{J_{pm}(q_0, u_0, n) J_{pm}(q_1, u_0, n)}{J_{pm}(q_0, u_0, n) J'_{pm}(q_1, u_0, n) - J_{pm}(q_1, u_0, n) J'_{pm}(q_0, u_0, n)}$$

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In the quasi-static limit \( q_0 = \frac{K_0^2 F^2}{4} \ll 1 \), \( q_1 = q_0 \varepsilon_r \ll 1 \), the closed-form condition for a PEC elliptical cylinder under TM-polarized illumination can be derived as:

\[
Z_{s-opt} = j \omega \mu F \cosh u_0 \frac{u_0 - u_1}{1 + \sinh 2u_0 \left( q_0 (u_0 - u_1) + q_1 \left( u_1 + \frac{1}{2} \ln q_1 \right) \right)}
\]

And also, the closed-form condition for a dielectric elliptical cylinder under TM-polarized illumination can be derived as:

\[
Z_{s-opt} = j \omega \mu F \frac{1 - q_1 \sinh^2 u_0}{2 \sinh u_0 (q_0 - q_1)}
\]
Frequency dispersion of the surface reactance for graphene monolayer and nanopatches with respect to the optimum required is found as:

- Required Reactance for Dielectric
- Required Reactance for PEC
- Required Reactance for Strip

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Surface Conductivity of Graphene

**Kubo Formula**

\[ \sigma(\omega, \mu_c, \tau, T) = -\frac{je^2(\omega+j\tau^{-1})}{\pi\hbar^2} \times \left[ \frac{1}{(\omega+j\tau^{-1})} \int_0^\infty \left( \frac{\partial f_d(\varepsilon)}{\partial \varepsilon} - \frac{\partial f_d(-\varepsilon)}{\partial \varepsilon} \right) \varepsilon d\varepsilon - \int_0^\infty \frac{f_d(-\varepsilon)-f_d(\varepsilon)}{(\omega+j\tau^{-1})^2-4(\varepsilon/\hbar)^2} d\varepsilon \right] \]

**Intraband Contributions**

\[ \sigma_{\text{intra}} = j \frac{e^2k_BT}{\pi\hbar^2(\omega+j\tau^{-1})} \left[ \frac{\mu_c}{k_BT} + 2\ln \left( e^{\frac{-\mu_c}{k_BT}} + 1 \right) \right] \]

\[ Z_s = \frac{1}{\sigma} \]

**Interband Contributions**

\[ \sigma_{\text{inter}} = \frac{je^2}{4\pi\hbar} \ln \left( \frac{2|\mu_c|-(\omega+j\tau^{-1})\hbar}{2|\mu_c|+(\omega+j\tau^{-1})\hbar} \right) \]

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\(-e\) : charge of electron  
\(\omega\) : angular frequency  
\(\mu_c\) : chemical potential  
\(T\) : temperature  
\(\varepsilon\) : energy  
\(\hbar\) : reduced Planck’s constant  
\(\tau\) : momentum relaxation time

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Dielectric Elliptical Cylinder at THz Frequencies

- Geometry parameters are:
  \[ \varepsilon_r = 4 \text{ (silicon dioxide)} \]
  \[ a_0 = 12.5 \mu m \left( \frac{\lambda_0}{8} \right) \]
  \[ b_0 = 10 \mu m \left( \frac{\lambda_0}{10} \right) \]

- The required reactance is found to be: \[ X_s = 260 \, \Omega \]

- The design parameters are: \[ \mu_c = 0.6158 \, \text{eV}, \quad T = 300 \, \text{K}, \quad \tau = 1.5 \, \text{ps} \]
Power Flow and Far-field Pattern

Uncloaked

Cloaked

\[ f = 3 \text{ THz} \]

\[ \varepsilon_r = 4 \text{ (silicon dioxide)} \]
\[ a_0 = 12.5 \mu m \left( \lambda_0 / 8 \right) \]
\[ b_0 = 10 \mu m \left( \lambda_0 / 10 \right) \]

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Electric Field Distribution

$\varepsilon_r = 4$ (silicon dioxide)
$a_0 = 12.5 \, \mu m \, (\lambda_0/8)$
$b_0 = 10 \, \mu m \, (\lambda_0/10)$

$f = 3 \, THz$

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Closely Spaced and Overlapping Dielectric Elliptical Cylinders

Uncloaked

Cloaked

Uncloaked

Cloaked

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Cluster of Dielectric Elliptical Cylinders I

Uncloaked

$g = 3 \, \mu m$

Cloaked

$l = 50 \, \mu m = 0.5 \lambda$

$f = 3 \, THz$

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Cluster of Dielectric Elliptical Cylinders II

Uncloaked

Cloaked

\( f = 3 \text{ THz} \)

\( g = 3 \mu m \)

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Overlapping of Dielectric Elliptical Cylinders

Uncloaked

Cloaked

$l = 140 \mu m = 1.4 \lambda$

$f = 3 \text{ THz}$

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Dielectric Elliptical Cylinder at Microwave Frequencies

- **Geometry parameters are:**
  \[
  \varepsilon_r = 10 \\
  a_0 = 10 \text{ mm } (\lambda_0/10) \\
  b_0 = 8.6655 \text{ mm } (\lambda_0/11.54)
  \]

- The required reactance is found to be: \( X_s = 70.75 \ \Omega \)

- The design parameters are:
  \[
  D = 7.339 \text{ mm } (\lambda_0/13.62), \ w = 0.3626 \text{ mm } (\lambda_0/275.786)
  \]

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Dielectric Elliptical Cylinder at Microwave Frequencies

- Geometry parameters are:
  \[ \varepsilon_r = 10 \]
  \[ a_0 = 10 \text{ mm (}\lambda_0/10\text{)} \]
  \[ b_0 = 8.6655 \text{ mm (}\lambda_0/11.54\text{)} \]

- The required reactance is found to be: \[ X_s = 70.75 \Omega \]

- The design parameters are:
  \[ N = 3 \]
  \[ D = 19.57 \text{ mm (}\lambda_0/5.109\text{)}, w = 4.9 \text{ mm (}\lambda_0/20.408\text{)} \]
  \[ D = 7.339 \text{ mm (}\lambda_0/13.62\text{)}, w = 0.3626 \text{ mm (}\lambda_0/275.786\text{)} \]

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Electric Field Distribution

Uncloaked

$N = 8$

$f = 3 \text{ GHz}$

$\varepsilon_r = 10$

$a_0 = 10 \text{ mm } (\lambda_0/10)$

$b_0 = 8.6655 \text{ mm } (\lambda_0/11.54)$

Cloaked

H. M. Bernety and A. B. Yakovlev, “Metasurface Cloaks for Dielectric and Metallic Elliptical Cylinders and Strips”

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Geometry parameters are:

- \( \varepsilon_r = 4 \) (silicon dioxide)
- \( a_0 = 12.5 \mu \text{m} \) (\( \lambda_0/8 \))
- \( b_0 = 10 \mu \text{m} \) (\( \lambda_0/10 \))

The required reactance is found to be: \( X_s = 260 \ \Omega \)

The design parameters are:

- \( D = 5.064 \mu \text{m} \)
- \( g = 0.524 \mu \text{m} \)
- \( \mu_c = 0.2672 \text{ eV} \)

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Geometry parameters are:
- $a_1 = 10.04 \, \mu m \left( \lambda_0/9.96 \right)$
- $b_1 = 6.67 \, \mu m \left( \lambda_0/15 \right)$

The required reactance is found to be: $X_s = -94.73 \, \Omega$

The design parameters are:
- $D = 5.064 \, \mu m$, $g = 0.524 \, \mu m$, $\mu_c = 0.5718 \, eV$
Electric Field Distribution

Uncloaked

Cloaked

$f = 3 \text{ THz}$

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Geometry parameters are:

- \( a_1 = 10 \text{ mm} (\lambda_0/10) \), \( b_1 = 8.865 \text{ mm} (\lambda_0/11.54) \)
- \( a_0 = 11.9164 \text{ mm} (\lambda_0/8.39) \), \( b_0 = 10.8167 \text{ mm} (\lambda_0/9.24) \)

The required reactance is found to be: \( X_s = -66.12 \Omega \)

The design parameters are:

- \( D = 7.145 \text{ mm} (\lambda_0/14) \), \( g = 0.99 \text{ mm} (\lambda_0/101.01) \)
2-D Metasurface Cloak for PEC at Microwaves

- Geometry parameters are:
  \[ a_1 = 10 \text{ mm} \left( \frac{\lambda_0}{10} \right), \quad b_1 = 8.865 \text{ mm} \left( \frac{\lambda_0}{11.54} \right) \]
  \[ a_0 = 11.9164 \text{ mm} \left( \frac{\lambda_0}{8.39} \right), \quad b_0 = 10.8167 \text{ mm} \left( \frac{\lambda_0}{9.24} \right) \]

- The required reactance is found to be: \( X_s = -66.12 \Omega \)

- The design parameters are:
  \( N = 10 \)
  \( D = 7.145 \text{ mm} \left( \frac{\lambda_0}{14} \right), \quad g = 0.99 \text{ mm} \left( \frac{\lambda_0}{101.01} \right) \)

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Power Flow and Far-field Pattern

Uncloaked

Cloaked

$N = 10$

$f = 3 \text{ GHz}$

$\theta$

$\phi$

$-0.938$

$-1.88$

$-2.81$

$-3.75$

$-4.69$

$-5.63$

$-6.56$

$-7.5$

$-8.44$

$-9.38$

$-10.3$

$-11.3$

$-12.2$

$-13.1$

$-14.1$

$-15$

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ICEAA 2014, Aruba
Uncloaked

$P_1 = 10 \text{ mm} \left( \frac{\lambda_0}{10} \right)$

$b_1 = 8.865 \text{ mm} \left( \frac{\lambda_0}{11.54} \right)$

$a_0 = 11.9164 \text{ mm} \left( \frac{\lambda_0}{8.39} \right)$

$b_0 = 10.8167 \text{ mm} \left( \frac{\lambda_0}{9.24} \right)$

$P_2 = 3 \text{ GHz}$

Cloaked

H. M. Bernety and A. B. Yakovlev, “Metasurface Cloaks for Dielectric and Metallic Elliptical Cylinders and Strips”
A strip can be modeled as a degenerated ellipse

Geometry parameters are:

\[ a_0 = 10.04 \, \mu m \left( \frac{\lambda_0}{9.96} \right), \quad b_0 = 6.67 \, \mu m \left( \frac{\lambda_0}{15} \right) \]
\[ a_1 = 7.5 \, \mu m = F \left( \frac{\lambda_0}{13.33} \right), \quad \varepsilon_r = 4 \]

\[ X_s = -210 \, \Omega \quad D = 5.29 \, \mu m, \quad g = 0.6 \, \mu m, \quad \mu_c = 0.9 \, eV \]
Power Flow and Far-field Pattern

Uncloaked

Cloaked

\( f = 3 \text{ THz} \)

H. M. Bernety and A. B. Yakovlev, “Metasurface Cloaks for Dielectric and Metallic Elliptical Cylinders and Strips”
Electric Field Distribution

H. M. Bernety and A. B. Yakovlev, “Metasurface Cloaks for Dielectric and Metallic Elliptical Cylinders and Strips”
Cloaking for Two Strips

H. M. Bernety and A. B. Yakovlev, “Metasurface Cloaks for Dielectric and Metallic Elliptical Cylinders and Strips”
Two Strips Horizontally Oriented

H. M. Bernety and A. B. Yakovlev, “Metasurface Cloaks for Dielectric and Metallic Elliptical Cylinders and Strips”
Two Strips with Overlapping Cloaks

Uncloaked

Cloaked

$\mathbf{f} = 3 \text{ THz}$

$g = 3.7 \mu \text{m}$

H. M. Bernety and A. B. Yakovlev, “Metasurface Cloaks for Dielectric and Metallic Elliptical Cylinders and Strips”
Two Connected Strips

Uncloaked

Cloaked

f = 3 THz

l = \lambda / 3.33

H. M. Bernety and A. B. Yakovlev, “Metasurface Cloaks for Dielectric and Metallic Elliptical Cylinders and Strips”
Conclusions

❖ An analytical approach has been proposed to cloak elliptical cylinders, and also, strips at microwave and low-THz frequencies by using conformal mantle cloak designs.

❖ Although the electromagnetic wave scattering of an elliptical cylinder is pertinent to the angle of incidence, it is shown that the cloak design is robust for any incident angle.

❖ The idea has been extended to different geometries such as clusters of dielectric elliptical cylinders and various cases of strips.
Conclusions

Challenges for future work???

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