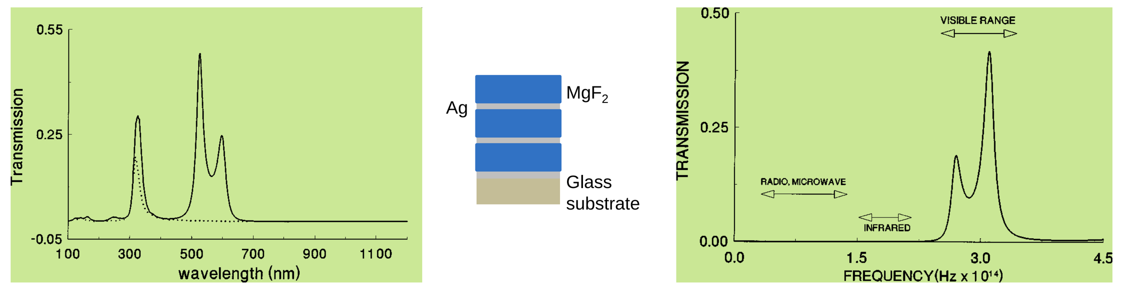
Sub-Wavelength Transmission through Stacked Two-Dimensional Metallic Patches: A Circuit Model Perspective

C. S. R. Kaipa, A. B. Yakovlev, F. Medina, F. Mesa, and Y. R. Padooru

Background and Motivation

Multilayer metal-dielectric stack at optical frequencies



Scalora et al., *Jour. App. Physics* , 83, 2377 (1998).

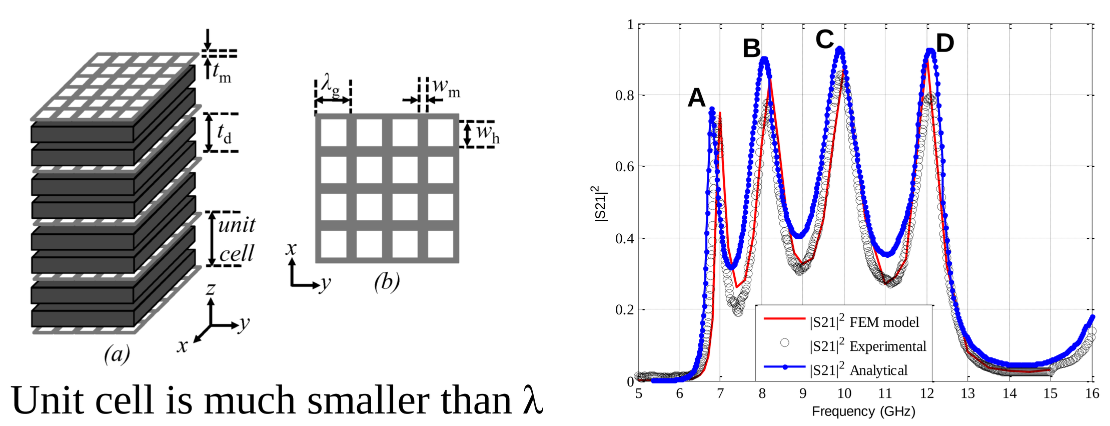
Enhanced Transmission

- ✓ Fabry-Perot Resonances
- ✓ PBG Theory

Transparency regions can be tuned by controlling the thickness of

- ✓ Dielectrics or semiconductors
- ✓ Metal films

Microwave transmissivity of a metamaterial-dielectric stack



Unit cell is much smaller than λ

At **microwaves** the **metal films** are substituted by **perforated metal layers**. **Butler et al.**, *Appl. Phys. Lett.* , 95, 174101 (2009)

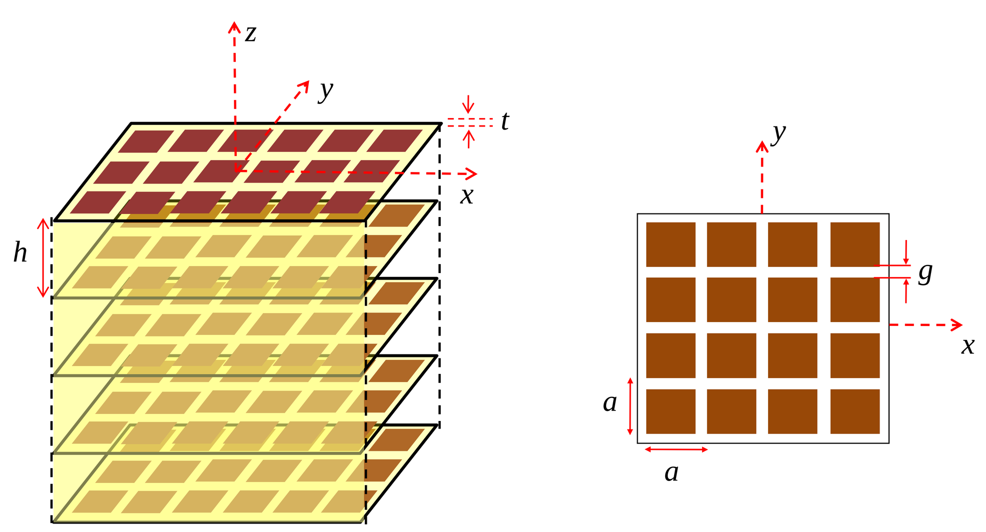
Stacked metallic mesh **grids** have been accurately analyzed using **circuit models**. **Kaipa et al.**, *Opt. Express.*, 18, 13309-13320 (2010)

Number of **transmission peaks** is equal to the number of aperture coupled transmission line sections.

The frequency of **mode D** is very close to the **Fabry-Perot** frequency of single slab.

Statement of the Problem

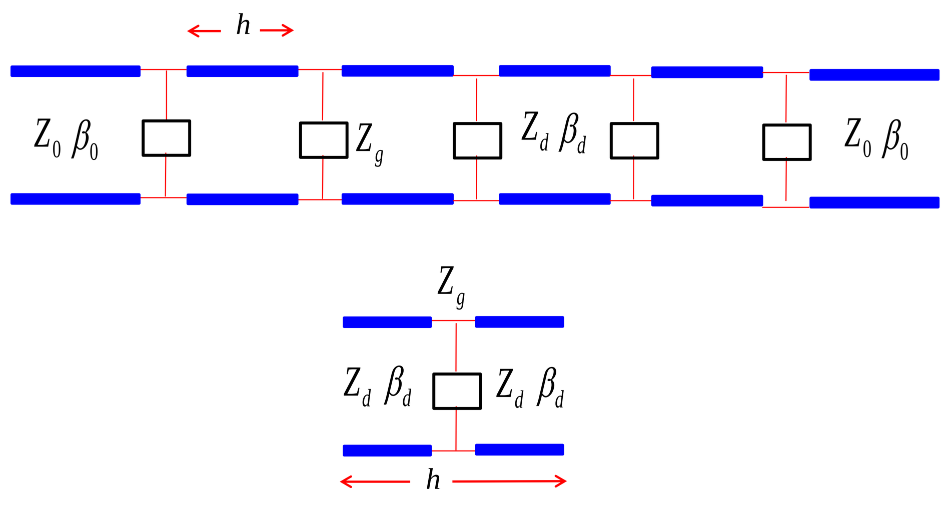
We are interested on the analysis of the transmission features of the stack of metal patches using **circuit model**



Unit cell is much smaller than λ

Circuit models provide **fast solvers** and simple **physical interpretation,** and have been used in the past to analyze metal grids **R. Ulrich**, *Infrared Physics*, 7, 37-55 (1967)

Circuit Modeling for Normal Incidence



Unit Cell

Transmission Line Parameters

$$\beta_0 = \frac{\omega}{c}; \quad \beta_d = \beta_0\sqrt{\varepsilon_r}\left(1 - j\tan\delta\right)$$

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}; \quad Z_d = \frac{Z_0}{\sqrt{\varepsilon_r\left(1 - j\tan\delta\right)}}$$

Grid Impedance

- ✓ **Dynamic model** for effective grid impedance of patch grid
- ✓ **Averaged impedance boundary condition**

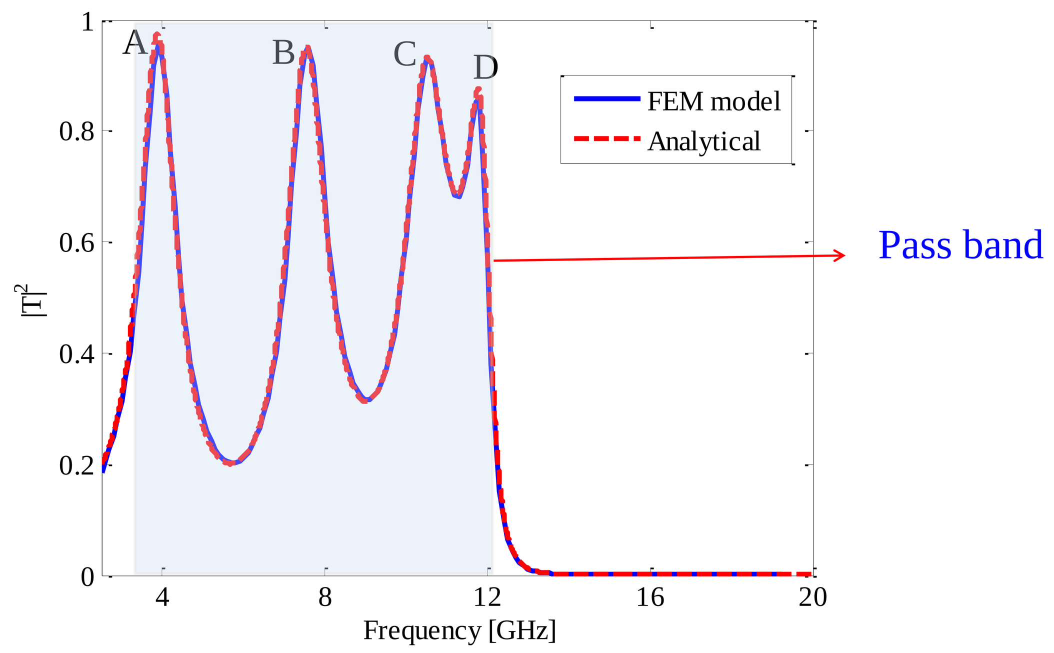
$$Z_g = \frac{a}{(a-g)\sigma} - j\frac{\eta_{eff}}{2\alpha}$$

$$\alpha = \frac{k_{eff}\,a}{\pi}\ln\left[\csc\left(\frac{\pi g}{2a}\right)\right] \qquad \sigma = \sigma_{3D}\times\delta_s$$

Luukkonen et al., *IEEE Trans. Antennas Propagat.*, 56, June 2008

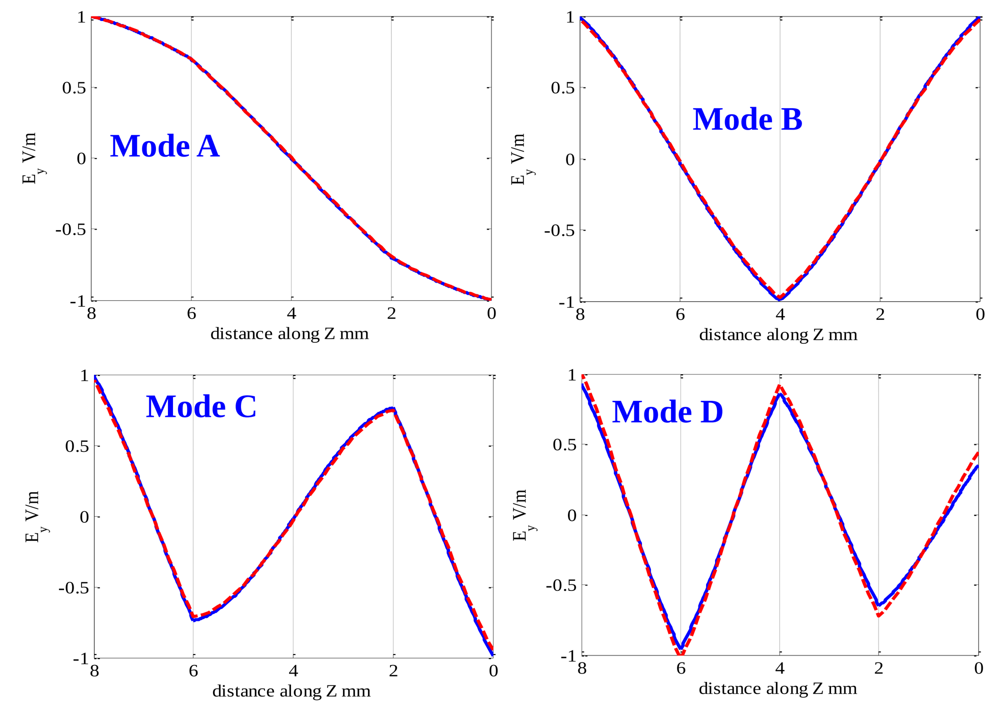
Circuit Model Results / HFSS Simulations

Substrate thickness (h) = 2 mm
Period (a) = 2 mm
Gap (g)= 0.2 mm,
Permittivity (ε_r) = 10.2,
Thickness of grid (t) = 18 μm
Loss tangent (tanδ) = 0.0035

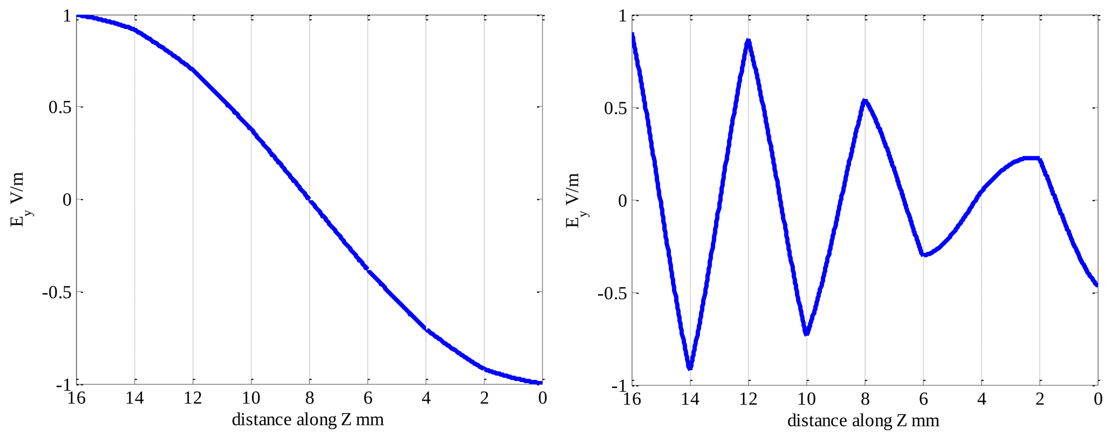


Number of **transmission peaks** is equal to the number of **transmission line sections** (capacitive coupled)

Field Distributions: Physics of the problem



Frequencies of interest are the ones corresponding to the **lower** and the **upper band edges.**



The **field pattern** for the **first** and the **last** resonance is of the same qualitative behavior.

Relevant Facts

- ✓ The frequency of mode A **(lower limit)** is very close to the **Fabry-Perot frequency** of entire **multilayer structure** ignoring the internal grids.
- ✓ The frequency of mode D **(upper limit)** is close to the **Fabry-Perot frequency** of the **single FP cavity**.
- ✓ The results can be interpreted in terms of "**excess lengths**" associated with two type of discontinuities: inner grids and outer grids.
- ✓ When the number of cells along z is increased the frequencies of the lower and upper band edges tend to a limit value.

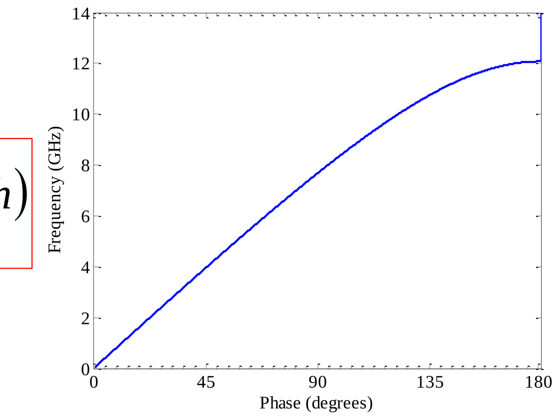
Frequencies with number of layers

No of Layers	f_{LB} (GHz)	f_{UB} (GHz)
4	3.88	11.77
5	3.126	11.8
6	2.616	11.82
10	1.578	11.9
18	0.882	11.99
30	0.528	12.02

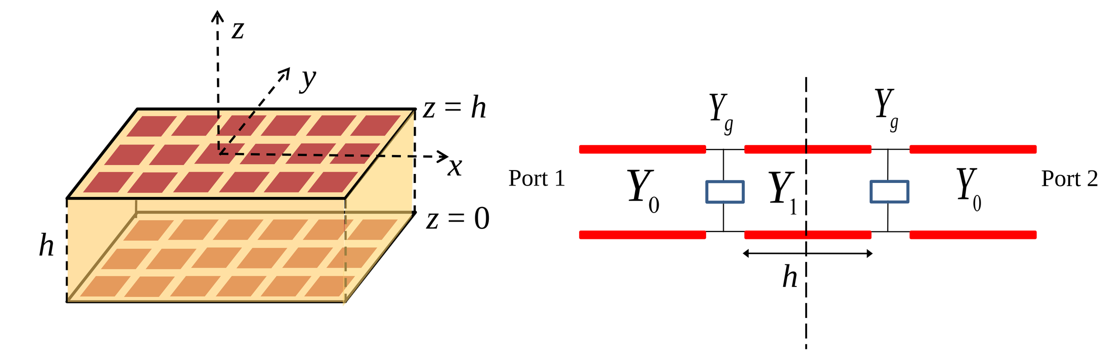
Propagation Characteristics

Dispersion equation for periodic structure

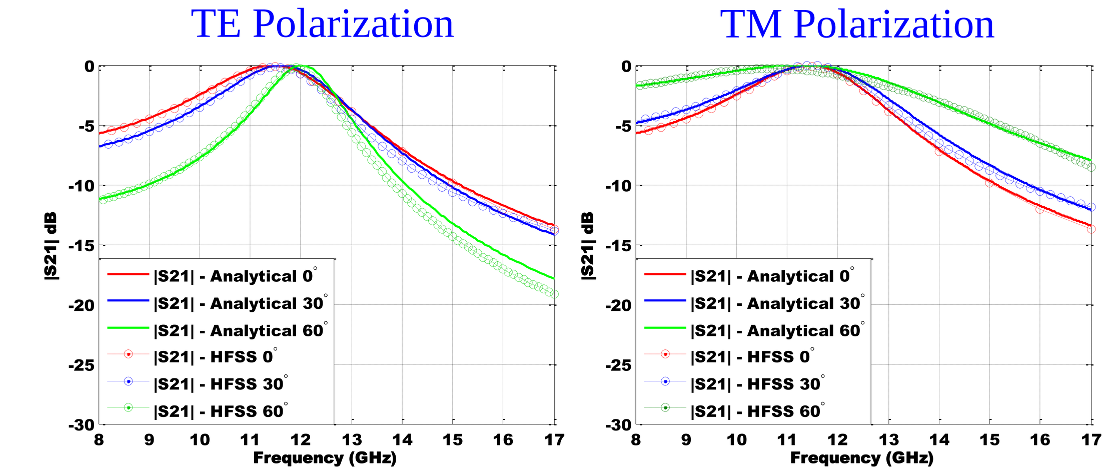
$$\cosh(\gamma h) = \cos(k_d h) + \frac{jZ_d}{2Z_g}\sin(k_d h)$$



Total Transmission: Excess Lengths



Substrate thickness (h): 2 mm
Period (a): 2 mm
Gap (g): 0.2 mm
Dielectric permittivity (ε_r): 10.2



Sub-wavelength resonances of complete transmission.

Fabry-Perot resonances is the underlying mechanism.

Even excitation

$$S_{11}^{e} = \frac{Y_0 - Y_g - jY_1\tan(\beta_1 h/2)}{Y_0 + Y_g + jY_1\tan(\beta_1 h/2)}$$

Odd excitation

$$S_{11}^{o} = \frac{Y_0 - Y_g + jY_1\cot(\beta_1 h/2)}{Y_0 + Y_g - jY_1\cot(\beta_1 h/2)}$$

Total transmission $|S_{11}| = 0$

$$\tan(\beta_1 h) = j\frac{2Y_1 Y_g}{Y_1^2 - Y_0^2 + Y_g^2}$$

Dispersion equation with roots ($\beta_1 h$) corresponding to total transmission **Medina et al.**, *IEEE Trans. Microw. Theory Tech.*, Jan. 2010

$$H(\beta_1 h) = \tan(\beta_1 h) - \frac{-2\omega Y_1 C_g}{Y_1^2 - Y_0^2 - (\omega C_g)^2} = 0$$

Dispersion equation for two-sided patch array (capacitive grid)

$$f_{TT}^{TE/TM} = \frac{c}{2\sqrt{\varepsilon_r - \sin^2\theta}\left(h + \Delta h^{TE/TM}\right)}$$

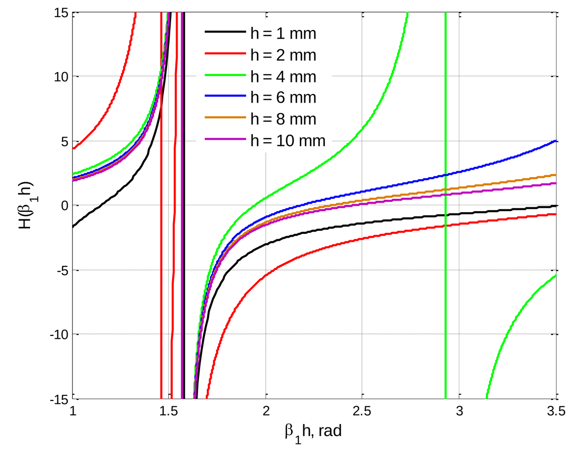
Frequency of **total transmission** for two-sided patch structure.

$$\Delta h^{TE} = \frac{2c\eta_0 C_g^{TE}}{\varepsilon_r - \sin^2\theta} \qquad \Delta h^{TM} = \frac{2c\eta_0 C_g^{TM}}{\varepsilon_r}$$

Excess length associated with edge capacitance.

Total transmission as a function of thickness (h)

h (mm)	f_{TT} (GHz) Calculated	f_{TT} (GHz) Excess length	f_{TT} (GHz) HFSS
1	17.2105	13.0606	16.68
2	11.425	10.2174	11.4
4	7.279	7.1182	7.23
6	5.458	5.4616	5.37
8	4.392	4.4304	4.38
10	3.686	3.7268	3.65



Conclusion

- The observed **resonances of transmission** at low frequencies in multilayered sub-wavelength grids correspond to **Fabry-Perot type resonances** of a dielectric slab loaded with effective grid admittances
- The **range of frequencies** where the peaks are expected for a finite stacked structure can be analytically and accurately estimated from the **Bloch analysis** using the proposed circuit model
- **Analytical formulas** for frequencies of **transmission** are obtained for two-sided patch arrays in terms of an **excess length** associated with effective edge capacitance