Modal Propagation on Thin Metal Mushroom-Type Surfaces in the Transition to Bed-of-Nails-Type Wire Media

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Outline

- Introduction

- Nonlocal Homogenization Model for Mushroom Surfaces with Thin Metal/Graphene Patches
  - Generalized Additional Boundary Condition

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**Introduction – Mushroom-Type HIS**

Additional boundary condition for the “microscopic” current at the via-ground plane connection ($y = 0^+$) and via-patch connection ($y = L^-$):

\[
\left. \frac{dI(y)}{dy} \right|_{y=0^+} = 0
\]

In terms of “macroscopic” field components:

\[
\left[ \beta \varepsilon_r \frac{dE_y}{dy} + k_z \eta_0 \frac{dH_x}{dy} \right]_{y=0^+} = 0
\]

In mushroom-type HIS structures with short vias, the presence of patches significantly reduces the spatial dispersion in the wire-medium slab resulting in the uniform current along the vias.


\[ a/L > 1 \]

\[ k_0 \sqrt{\varepsilon_r} L << \pi \]
Nonlocal and Local Homogenization Models

The results of natural modes [surface waves and leaky waves] based on non-local (SD+ABC) and local (ENG) homogenization models are in excellent agreement

A. B. Yakovlev et al., IEEE Trans. Microwave Theory Tech., 57, Nov. 2009
Mushroom HIS – Thin Metal Patch

Metal patches

TM polarization: 30°

\[ \sigma_{3D} = 5.8 \times 10^7 \text{ (S/m)} \]

Metal Thickness: 1nm

**Generalized Additional Boundary Condition (GABC)**

**Thin Metal/Graphene Patch**

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**Leontovitch Boundary Condition:**

\[
\vec{E}_t = \frac{\eta_0 \mathbf{J}_c}{\sqrt{\varepsilon_{r,metal}}} \\
\varepsilon_{r,metal} = 1 - j \frac{\sigma_{3D}}{\omega \varepsilon_0}
\]

**Good conductor**

**Thin metal**

\[
\vec{E}_t = \frac{1}{\sigma_{2D}} \mathbf{J}_c \\
\sigma_{2D} = \sigma_{3D} t
\]

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**Surface Conductivity of Graphene**

\[
\sigma_{2D}^{\text{intra}} = - j \frac{e^2 k_B T}{\pi \hbar^2 (\omega - j 2 \Gamma)} \left( \frac{\mu_c}{k_B T} + 2 \ln \left( e^{-\mu_c / k_B T} + 1 \right) \right)
\]

- \(e\): charge of an electron
- \(k_B\): Boltzmann’s constant
- \(\mu_c\): chemical potential (electrostatic bias)
- \(\Gamma\): electron scattering rate (10^{12} Hz)
- \(T\): temperature (300 K)
- \(\hbar\): modified Planck’s constant

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**References**

G. W. Hanson, *J. Appl. Phys.*, 103, 2008

**Generalized Additional Boundary Condition (GABC)**

**Thin Metal/Graphene Patch**

**Surface charge density** for thin metal/graphene at the connection point, $y = L^-$:

$$\rho_s = \varepsilon_0 \varepsilon_r \hat{n} \cdot \vec{E} \quad \Rightarrow \quad \rho_s = \frac{\varepsilon_0 \varepsilon_r}{\sigma_{2D}} \mathcal{J}_c$$

**Principle of Conservation of Surface Charge**

$$\rho_s = - \frac{\nabla_s \cdot \vec{J}_c}{j \omega} \quad \Rightarrow \quad \rho_s = - \frac{1}{j \omega} \frac{dJ_c(y)}{dy} \quad \text{Thin wires}$$

**Generalized Additional Boundary Condition (GABC)** at the via-thin metal/graphene connection, $y = L^-$:

$$\left[ \frac{\sigma_{2D}}{j \omega \varepsilon_0 \varepsilon_r} \frac{dJ_c(y)}{dy} + J_c(y) \right]_{y=L^-} = 0$$

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Nonlocal Model – SD + ABC

- Wire medium slab as uniaxial continuous material characterized by tensor effective permittivity
- Spatial dispersion

\[
\tilde{\varepsilon}_{\text{eff}} = \varepsilon_0 \varepsilon_r \left( \varepsilon_{xx} \hat{x}\hat{x} + \hat{y}\hat{y} + \hat{z}\hat{z} \right)
\]

\[
\varepsilon_{xx} = 1 - \frac{\beta_p^2}{\beta_r^2 - k_x^2}
\]

\[
\beta_r = \beta \sqrt{\varepsilon_r}
\]

\[
\beta = \omega / c
\]

\[
\beta_p \text{ is the plasma wavenumber}
\]

\[
\beta_p^2 = \frac{2\pi / a^2}{\ln \left( \frac{a}{2\pi r_0} \right) + 0.5275}
\]

SD + ABC Model

• Magnetic field of TM\(^2\) natural modes:

\[
H_y = Ae^{-\gamma_0 x} e^{-jk_z z} \quad \text{air region}
\]
\[
H_y = \left( B_{TM} \cos(\beta_r x) + B_{TM} \cosh(\gamma_{TM} x) \right) e^{-jk_z z} \quad \text{wire medium slab}
\]

• Two-sided impedance boundary condition at \( x = L \):

\[
E_z \big|_{x=L^-} = E_z \big|_{x=L^+} = Z_g \left( H_y \big|_{x=L^+} - H_y \big|_{x=L^-} \right)
\]

• Generalized additional boundary condition at the via-thin metal/graphene patch connection, \( x = L^- \):

\[
\left[ \frac{\sigma_{2D}}{j\omega\varepsilon_0\varepsilon_r} \frac{dJ_c(x)}{dx} + J_c(x) \right] \big|_{x=L^-} = 0
\]

In terms of field components:

\[
\left[ \frac{\sigma_{2D}}{j\omega\varepsilon_0\varepsilon_r} \left( \beta\varepsilon_r \frac{dE_x}{dx} - k_z \eta_0 \frac{dH_y}{dx} \right) \left( \beta\varepsilon_r E_x - k_z \eta_0 H_y \right) \right] \big|_{x=L^-} = 0
\]

• Additional boundary condition at the via-ground plane connection, \( x = 0^+ \):

\[
\left( \frac{dJ_c(x)}{dx} \right) \big|_{x=0^+} = 0
\]

In terms of field components:

\[
\left[ \beta\varepsilon_r \frac{dE_x}{dx} - k_z \eta_0 \frac{dH_y}{dx} \right] \big|_{x=0^+} = 0
\]
Grid Impedance of Thin Metal/Graphene Patches

\[ Z_g = \frac{a}{(a - g)\sigma_{2D}} - j \frac{\eta_{\text{eff}}}{2\alpha} \]

\[ \alpha = \frac{k_{\text{eff}}a}{\pi} \ln \left( \csc \left( \frac{\pi g}{2a} \right) \right) \]

\[ \eta_{\text{eff}} = \frac{\eta_0}{\sqrt{\varepsilon_{\text{eff}}}} \quad k_{\text{eff}} = k_0\sqrt{\varepsilon_{\text{eff}}} \]

\[ \varepsilon_{\text{eff}} = \frac{\varepsilon_r + 1}{2} \]

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Dispersion Equation

- **TM**^2** natural modes

\[
H(k_z, \omega) = K \coth(\gamma_{TM} L) \cot(\beta_r L) + \left( \frac{1}{\gamma_0} - j \frac{\eta_0}{Z_g k_0} \right) = 0
\]

where

\[
K = \frac{1}{\varepsilon_{xx}^TM} - 1 \left( \frac{\sigma_{2D} \gamma_{TM}}{j \omega \varepsilon_0 \varepsilon_r} \tanh(\gamma_{TM} L) + 1 \right) + \left( 1 - \frac{\sigma_{2D} \beta_r}{j \omega \varepsilon_0 \varepsilon_r} \tan(\beta_r L) \right)
\]

\[
- \frac{\beta_r}{\varepsilon_r} \left( \frac{1}{\varepsilon_{xx}^TM} - 1 \right) \left( \frac{\sigma_{2D} \gamma_{TM}}{j \omega \varepsilon_0 \varepsilon_r} + \coth(\gamma_{TM} L) \right) + \frac{\gamma_{TM}}{\varepsilon_r} \left( \cot(\beta_r L) - \frac{\sigma_{2D} \beta_r}{j \omega \varepsilon_0 \varepsilon_r} \right)
\]

\[
\varepsilon_{xx}^TM = 1 - \frac{\beta_p^2}{k_z^2 + \beta_p^2}
\]

\[
\gamma_{TM} = \sqrt{\beta_p^2 + k_z^2 - \beta_r^2}
\]

\[
\gamma_0 = \sqrt{k_z^2 - \beta^2}
\]

In the limiting case \(\sigma_{2D} \rightarrow 0\) (transparent patches) it turns to wire-medium slab:


In the limiting case \(\sigma_{2D} \rightarrow \infty\) (PEC patches) it turns to mushroom HIS:

Mushroom Surface with Metallic Patches

**TM^z** natural modes

- Period: 2 mm
- Gap: 0.2 mm
- Radius of vias: 0.05 mm
- Slab thickness: 1 mm
- Dielectric permittivity: 10.2

\[ \sigma_{3D} = 5.8 \times 10^7 \text{ (S/m)} \]

\[ \sigma_{2D} = \sigma_{3D} t \]

\[ t \ll \delta, \quad \delta = \sqrt{2/\omega \mu_0 \sigma_{3D}} \]
Mushroom HIS – PEC Patches

Transition from proper to improper complex leaky waves: from backward to forward radiation

Mushroom HIS – Thin Metallic Patches

Proper complex and improper complex leaky waves of mushroom HIS parameterized by metal thickness

\[ \sigma_{3D} = 5.8 \times 10^7 \text{ (S/m)} \]

\[ \sigma_{2D} = \sigma_{3D} \times t \]

Proper complex and improper complex leaky waves are significantly perturbed with varying the metal thickness. With decrease in the metal thickness the leaky waves attenuate rapidly before radiating.
Transition of Mushroom HIS to WM Slab

Leaky Waves

Phase constant

Wire-medium slab

\[ \sigma_{3D} = 5.8 \times 10^7 \, \text{(S/m)} \]

\[ \sigma_{2D} = \sigma_{3D} \, t \]

solid lines: proper complex

dashed lines: improper complex

Physical leaky waves of the mushroom HIS structure in the transition to the wire-medium slab continue to physical leaky waves of wire-medium slab

A. B. Yakovlev et.al., IEEE Trans. Microwave Theory Tech., 57, Nov. 2009
Physical leaky waves of the mushroom HIS structure in the transition to the wire-medium slab continue to physical leaky waves of wire-medium slab

\[ \sigma_{3D} = 5.8 \times 10^7 \, \text{S/m} \]
\[ \sigma_{2D} = \sigma_{3D} t \]

Mushroom HIS – Thin Metallic Patches

Proper complex and improper complex solutions of mushroom HIS parameterized by metal thickness

Metal thickness: 100 nm

solid lines: proper complex
dashed lines: improper complex
Mushroom HIS – Thin Metallic Patches

**Proper complex** and **improper complex** solutions of mushroom HIS parameterized by metal thickness

**Metal thickness: 10 nm**

solid lines: **proper complex**
dashed lines: **improper complex**
Mushroom HIS – Thin Metallic Patches

Phase constant

Metal thickness: 2 nm

Metal thickness: 1 nm

solid lines: proper complex

dashed lines: improper complex
Mushroom HIS – Thin Metallic Patches

Attenuation constant

Metal thickness: 2 nm

Metal thickness: 1 nm

solid lines: proper complex
dashed lines: improper complex
Mushroom HIS – Modal Interaction (1)

Phase constant

Metal thickness: 1.08 nm

Metal thickness: 1.07 nm

solid lines: proper complex
dashed lines: improper complex
Mushroom HIS – Modal Interaction (1)

Attenuation constant

Metal thickness: 1.08 nm

Metal thickness: 1.07 nm

Solid lines: proper complex
Dashed lines: improper complex
Mushroom HIS – Modal Interaction (1)

Pole dynamics

Metal thickness: 1.08 nm

Metal thickness: 1.07 nm

solid lines: proper complex
dashed lines: improper complex

Modal interaction (modal exchange) occurs with varying metal thickness (surface conductivity)
Mushroom HIS – Modal Interaction (2)

Pole dynamics

Metal thickness: 0.68 nm

Metal thickness: 0.65 nm

solid lines: proper complex
dashed lines: improper complex

Modal interaction (modal exchange) occurs with varying metal thickness (surface conductivity)
Complex Frequency-Plane Branch Points

- **Type 0 branch points** $\omega_{n}^{(0)}$ - positive and negative solutions of dispersion equation meet forming a second-order zero

- **Type 1 branch points** $\omega_{n}^{(1)}$ - separate branches of complex-conjugate solutions of dispersion equation (leaky-wave cutoff)

- **Type 2 branch points** $\omega_{n/n+2}^{(2)}$ - connect $n$ and $n+2$ different modes within a given class (TE or TM)

Complex frequency-plane branch points:

\[
H_{k_z}, \omega = H'_{k_z}, k_z, \omega = 0
\]

\[
H'_{\omega}, k_z, \omega, H''_{k_z k_z}, k_z, \omega \neq 0
\]


A complete rotation about $\omega_{n/n+2}^{(2)}$ in the complex frequency plane results in the smooth interchange of the $n$ and $n+2$ TM (TE) modes of the metamaterial structure
Perturbed Wire-Medium Slab

Phase constant

$$\sigma_{3D} = 5.8 \times 10^7 \text{ (S/m)} \quad \sigma_{2D} = \sigma_{3D} t$$

Wire-medium slab

solid lines: proper complex
dashed lines: improper complex

Proper complex solution of mushroom HIS with thin metallic patches in the limiting case (transparent patches) continues to a proper real (bound) mode of the wire-medium slab

A. B. Yakovlev et al., IEEE Trans. Microwave Theory Tech., 57, Nov. 2009
\[ \sigma_{3D} = 5.8 \times 10^7 \text{ (S/m)} \]
\[ \sigma_{2D} = \sigma_{3D} t \]

**Phase constant**

solid lines: **proper complex**
dashed lines: **improper complex**

**Wire-medium slab**

Proper real solution of perturbed wire-medium slab continues to a nonphysical proper complex mode

Perturbed Wire-Medium Slab

Attenuation constant

\[ \sigma_{3D} = 5.8 \times 10^7 \text{ (S/m)} \]
\[ \sigma_{2D} = \sigma_{3D} t \]

Wire-medium slab

solid lines: proper complex
dashed lines: improper complex

Proper real solution of perturbed wire-medium slab continues to a nonphysical proper complex mode

Conclusions

- A **Nonlocal (SD+ABC)** homogenization model with **generalized ABC** is proposed for the analysis of TM natural waves of mushroom structures with thin metal/graphene patches

- **Modal propagation of surface waves and leaky waves** is studied in the transition of mushroom-type surface to bed-of-nails-type wire media

- It is observed that for some values of surface conductivity (metal thickness) **modal interactions (modal exchange)** occur, which can be explained by the evolution of **complex frequency-plane branch-point singularities** across the real frequency axis