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Negative Refraction by a Multilayered Mushroom-type Metamaterial

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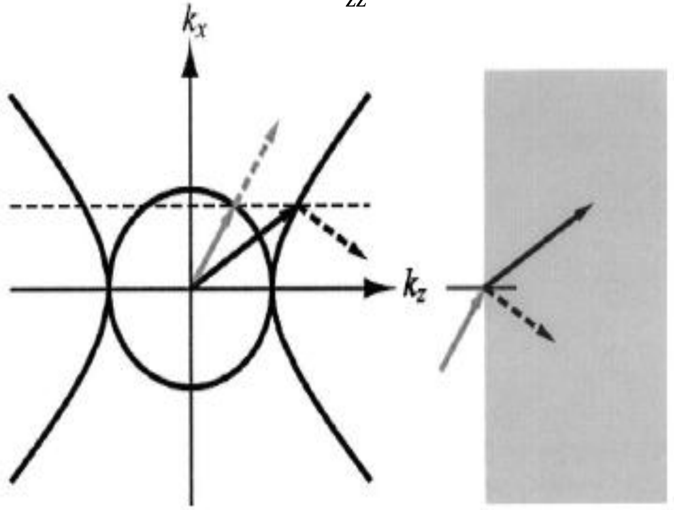
Motivation:

Negative refraction with indefinite media

$$\varepsilon_{xx} > 0$$

$$\varepsilon_{zz} < 0$$



$$\bar{\bar{\varepsilon}} = \begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix}$$

D. R. Smith and D. Schurig, Phys. Rev. Lett. 90, 077405 (2003)

D. R. Smith, D. Schurig, J. J. Mock, P. Kolinko, and P. Rye, APL 84, 2244, 2004

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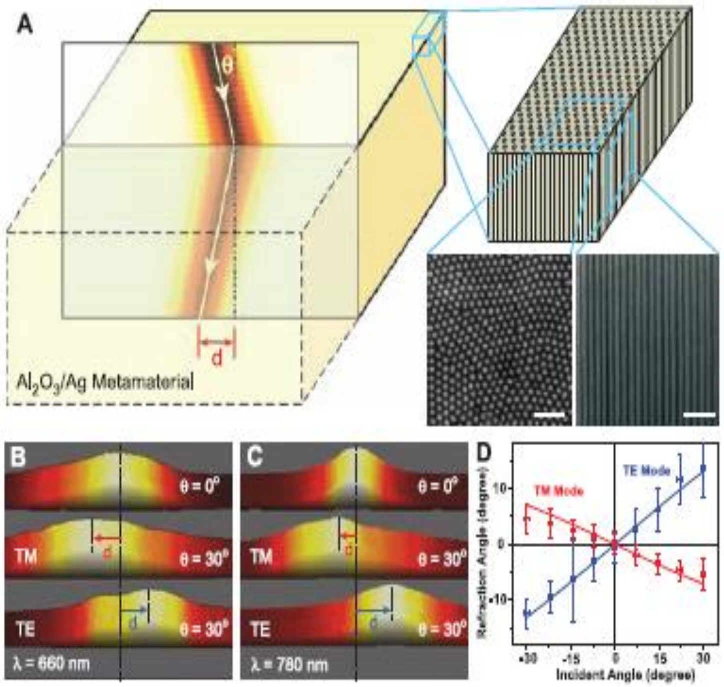
Negative refraction with arrays of nanorods in the visible:



J. Yao, Z. Liu, Y. Liu, Y. Wang, C. Sun, G. Bartal, A. M. Stacy,and X. Zhang, Science 321, 930, 2008.

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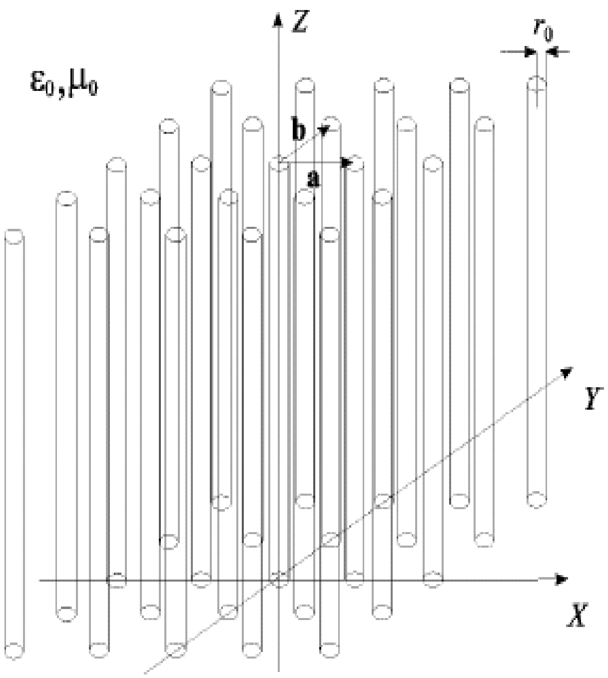
Can a similar effect be observed at microwave frequencies?



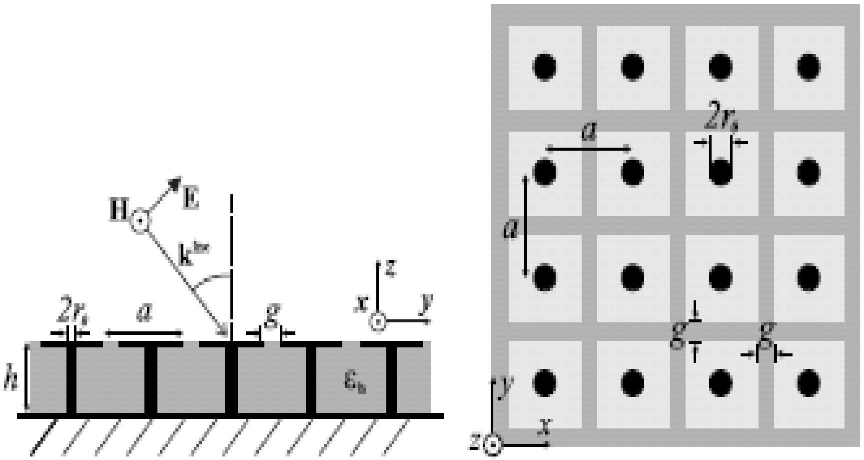
$$\varepsilon(k, q_z) = \varepsilon_0 \left(1 - \frac{k_0^2}{k^2 - q_z^2} \right)$$

P.A. Belov, R. Marqués, S. I. Maslovski, I.S. Nefedov, M. Silveirinha, C. R. Simovsky, S. A. Tretyakov, Phys. Rev. B 67, 113103 (2003).

An array of parallel rods does not behave as a local uniaxial ENG material at microwaves…

Taming the spatial dispersion

A Demetriadou and J B Pendry, J. Phys.: Condens. Matter 20 (2008) 295222



O. Luukkonen, M. G. Silveirinha, A. B. Yakovlev, C. R. Simovski, I. S. Nefedov, and S. A. Tretyakov, “Effects of Spatial Dispersion on Reflection from Mushroom-type Artificial Impedance Surfaces”, arxiv:0812.1658v1

A. B. Yakovlev, M. G. Silveirinha, O. Luukkonen, C. R. Simovski, I. S. Nefedov, and S. A. Tretyakov, “Homogenization Models for the Characterization of Surface-Wave Propagation on Mushroom Structures”.

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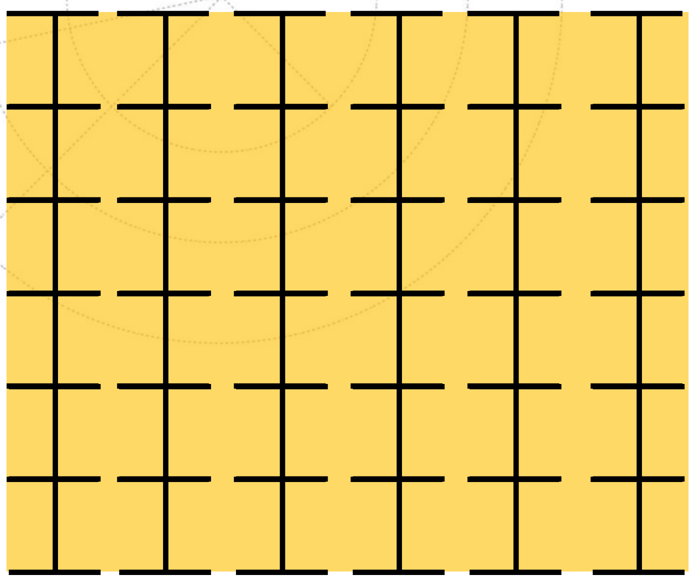
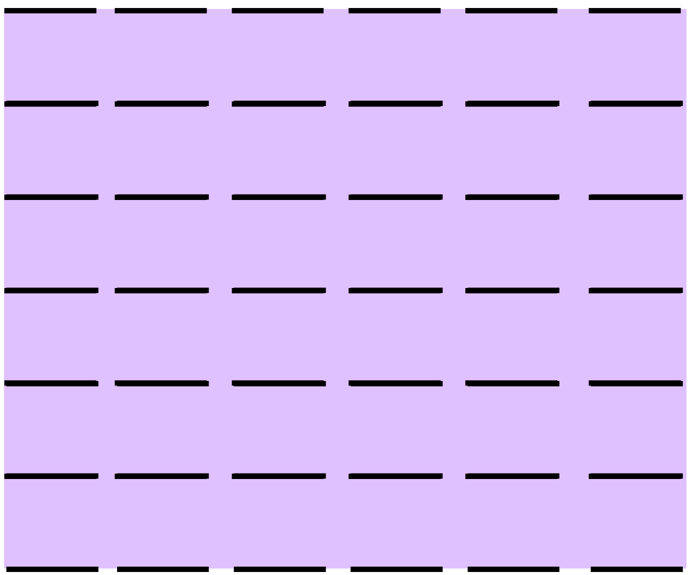
Hypothetical equivalence

Multilayered Mushroom



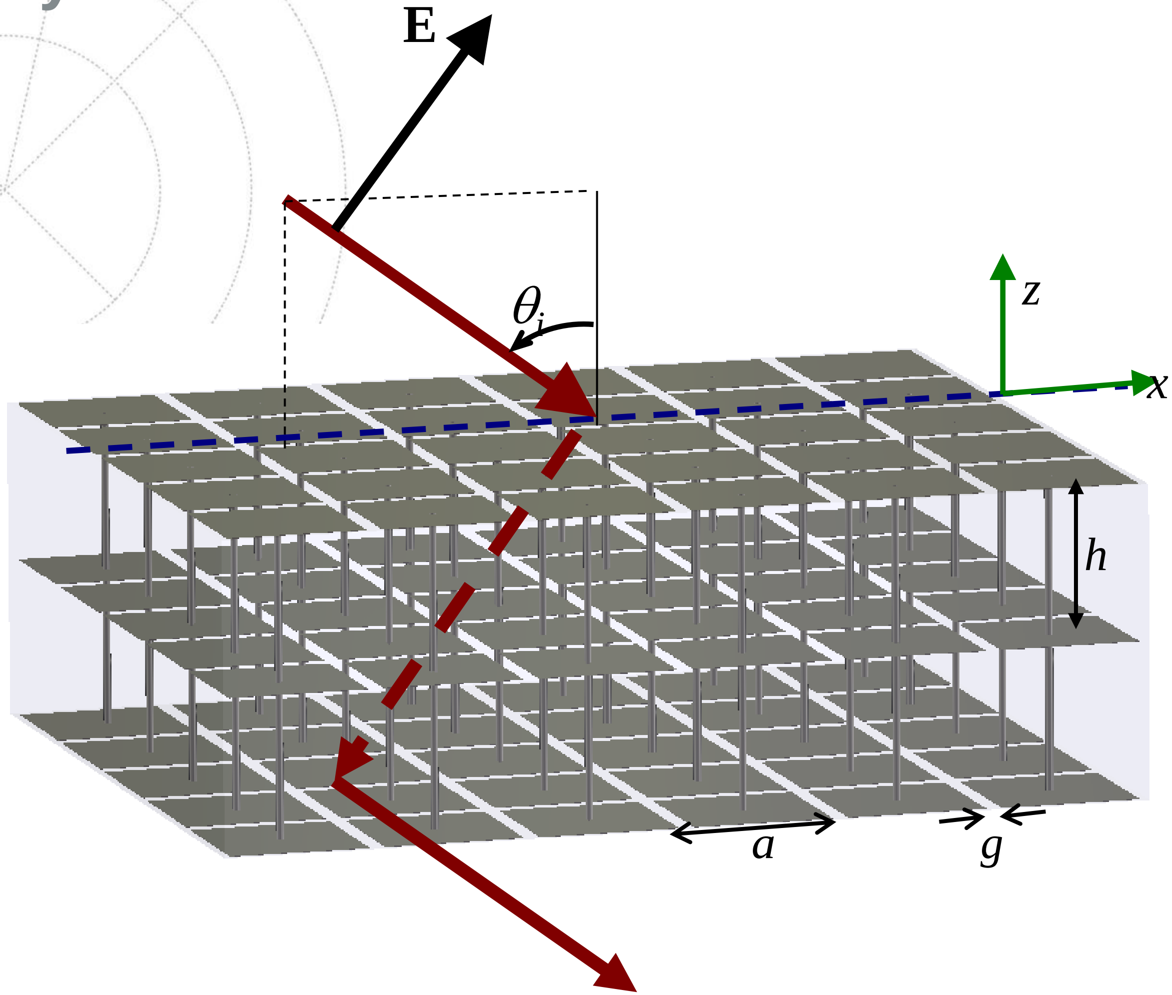
Local uniaxial ENG periodically loaded with patch grids



$$\bar{\bar{\varepsilon}} = \varepsilon_t \left(\hat{\mathbf{u}}_x \hat{\mathbf{u}}_x + \hat{\mathbf{u}}_y \hat{\mathbf{u}}_y \right) + \varepsilon_{zz} \hat{\mathbf{u}}_z \hat{\mathbf{u}}_z$$

$$\varepsilon_{zz} = \varepsilon_h - \frac{k_p^2}{k_0^2}$$

Geometry:



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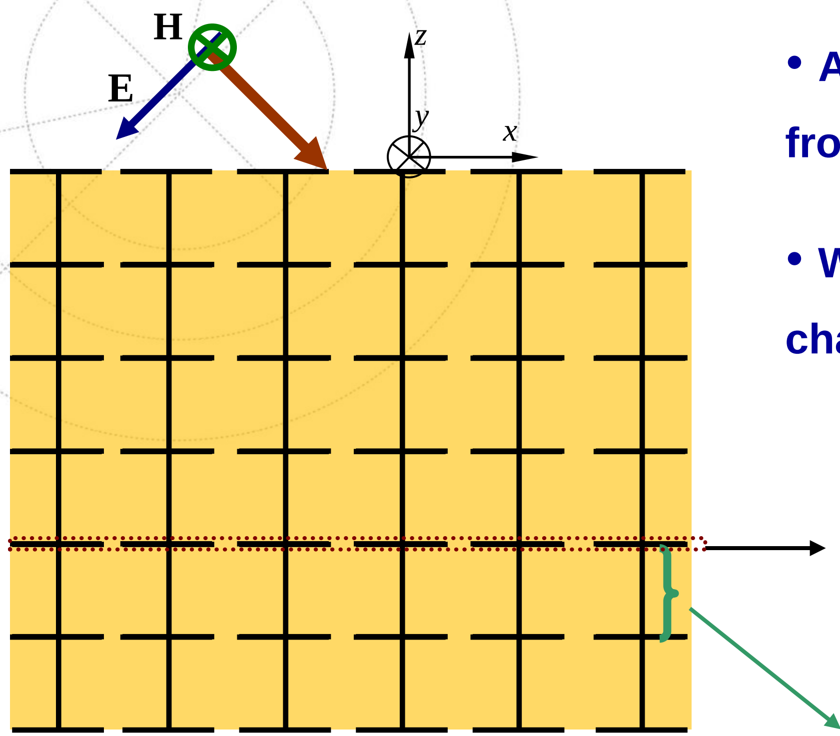
Scattering problem: Analytical model



- All the field components can be obtained from H_y
- We use a transfer matrix approach to characterize the reflected/transmitted waves

Transfer Matrix: $\mathbf{Q}$

Transfer Matrix: $\mathbf{M}$

Global Transfer Matrix: $\mathbf{M}_{\text{global}} = \mathbf{Q}_{\mathbf{av}}.\mathbf{M}...\mathbf{Q}.\mathbf{M}.\mathbf{Q}.\mathbf{M}.\mathbf{Q}_{\mathbf{av}}$

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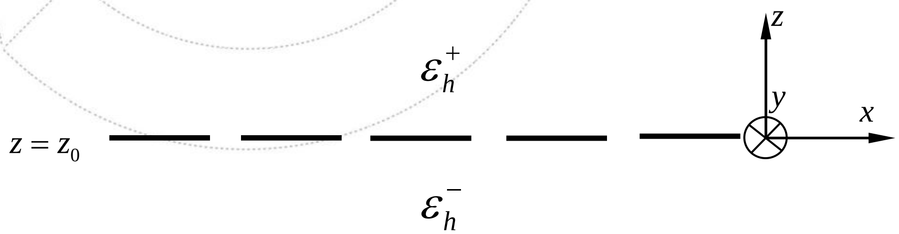
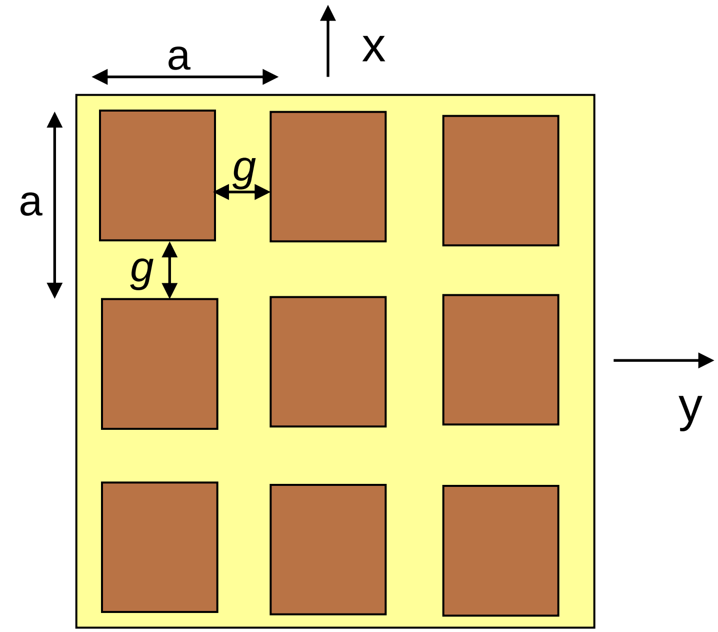
Transfer matrix for the patch grid:

$$\begin{pmatrix} E_x \\ \eta_0 H_y \end{pmatrix}_{z=z_0^+} = \mathbf{Q}.\begin{pmatrix} E_x \\ \eta_0 H_y \end{pmatrix}_{z=z_0^-}$$





$$\mathbf{Q} = \begin{pmatrix} 1 & 0 \\ -y_g & 1 \end{pmatrix} \quad \textbf{with} \quad y_g = \frac{Y_g}{Y_0} = \frac{\varepsilon_h^- + \varepsilon_h^+}{2} \frac{2a}{\pi} jk_0 \ln\left(\csc\left(\frac{\pi g}{2a}\right)\right)$$

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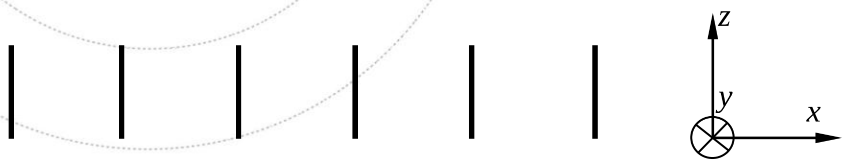
Transfer matrix for the wire medium slab:

$$\begin{pmatrix} E_x \\ \eta_0 H_y \end{pmatrix}_{z=z_0+h} = \mathbf{M}.\begin{pmatrix} E_x \\ \eta_0 H_y \end{pmatrix}_{z=z_0}$$

- **Field in WM-slab is written in terms of TM and TEM modes** **(coming from the nonlocal dielectric function)**



$$\eta_0 H_y = A_{TM}^{+} e^{-\gamma_{TM} z} + A_{TM}^{-} e^{+\gamma_{TM} z} + B_{TEM}^{+} e^{-\gamma_{TEM} z} + B_{TEM}^{-} e^{+\gamma_{TEM} z}$$

$$E_x = \frac{1}{\varepsilon_h k_0}\left[\left(-j\gamma_{TM}\right)\left(A_{TM}^{+} e^{-\gamma_{TM} z} - A_{TM}^{-} e^{+\gamma_{TM} z}\right) + \left(-j\gamma_{TEM}\right)\left(B_{TEM}^{+} e^{-\gamma_{TEM} z} + B_{TEM}^{-} e^{+\gamma_{TEM} z}\right)\right]$$

$$\gamma_{TEM} = jk_0\sqrt{\varepsilon_h} \qquad \gamma_{TM} = \sqrt{k_p^2 + k_x^2 - k_0^2\varepsilon_h}$$

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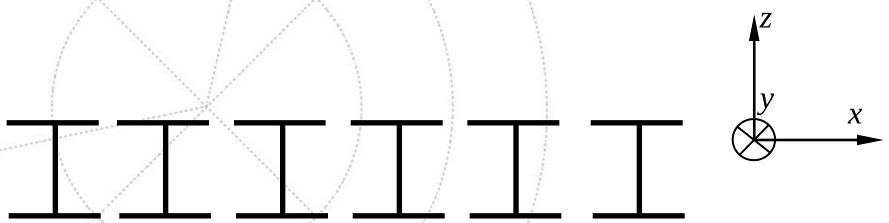
Transfer matrix for the wire medium slab (contd.):



Because the wires are actually connected to the patches, the amplitudes of the TEM and TM modes are not independent!

$$\eta_0 H_y = A_{TM}^{+} e^{-\gamma_{TM} z} + A_{TM}^{-} e^{+\gamma_{TM} z} + B_{TEM}^{+} e^{-\gamma_{TEM} z} + B_{TEM}^{-} e^{+\gamma_{TEM} z}$$

Additional boundary condition: $$\left. k_0 \varepsilon_h \frac{dE_z}{dz} + k_x \eta_0 \frac{dH_y}{dz} \right|_{\substack{z=z_0 \\ \text{and} \\ z=z_0+h}} = 0$$

M. Silveirinha, C. A. Fernandes, J. R. Costa, "Additional Boundary Condition for a Wire Medium Connected to a Metallic Surface", New J. Phys., 10, 053011(1-17), 2008

O. Luukkonen, M. G. Silveirinha, A. B. Yakovlev, C. R. Simovski, I. S. Nefedov, and S. A. Tretyakov, “Effects of Spatial Dispersion on Reflection from Mushroom-type Artificial Impedance Surfaces”, arxiv:0812.1658v1

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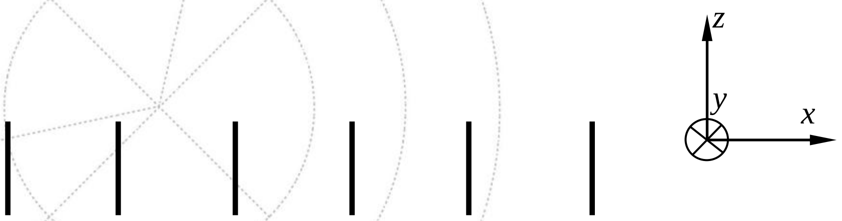
Transfer matrix for the wire medium slab (contd.):



$$\begin{pmatrix} E_x \\ \eta_0 H_y \end{pmatrix}_{z=z_0+h} = \mathbf{M}.\begin{pmatrix} E_x \\ \eta_0 H_y \end{pmatrix}_{z=z_0}$$

$$m_{11} = m_{22} = \frac{\left(\varepsilon_h - \varepsilon_{zz}^{TM}\right)\gamma_{TM}\sinh\left(\gamma_{TM}h\right)\cosh\left(\gamma_{TEM}h\right) + \varepsilon_{zz}^{TM}\gamma_{TEM}\cosh\left(\gamma_{TM}h\right)\sinh\left(\gamma_{TEM}h\right)}{\left(\varepsilon_h - \varepsilon_{zz}^{TM}\right)\gamma_{TM}\sinh\left(\gamma_{TM}h\right) + \varepsilon_{zz}^{TM}\gamma_{TEM}\sinh\left(\gamma_{TEM}h\right)}$$

$$m_{12} = \frac{1}{k_0}\frac{j\gamma_{TEM}\gamma_{TM}\sinh\left(\gamma_{TM}h\right)\sinh\left(\gamma_{TEM}h\right)}{\left(\varepsilon_h - \varepsilon_{zz}^{TM}\right)\gamma_{TM}\sinh\left(\gamma_{TM}h\right) + \varepsilon_{zz}^{TM}\gamma_{TEM}\sinh\left(\gamma_{TEM}h\right)}$$

$$m_{21} = \left(-jk_0\right)\frac{2\left(\varepsilon_h - \varepsilon_{zz}^{TM}\right)\varepsilon_{zz}^{TM}\left[-1 + \cosh\left(\gamma_{TM}h\right)\cosh\left(\gamma_{TEM}h\right)\right] + \sinh\left(\gamma_{TEM}h\right)\sinh\left(\gamma_{TM}h\right)\left[\left(\varepsilon_h - \varepsilon_{zz}^{TM}\right)^2\frac{\gamma_{TM}}{\gamma_{TEM}} + \left(\varepsilon_{zz}^{TM}\right)^2\frac{\gamma_{TEM}}{\gamma_{TM}}\right]}{\left(\varepsilon_h - \varepsilon_{zz}^{TM}\right)\gamma_{TM}\sinh\left(\gamma_{TM}h\right) + \varepsilon_{zz}^{TM}\gamma_{TEM}\sinh\left(\gamma_{TEM}h\right)}$$

$$\varepsilon_{zz}^{TM} = \varepsilon_h\left(1 - \frac{k_p^2}{k_x^2 + k_p^2}\right)$$

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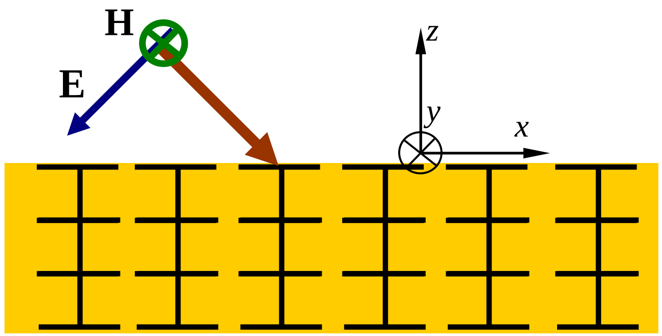
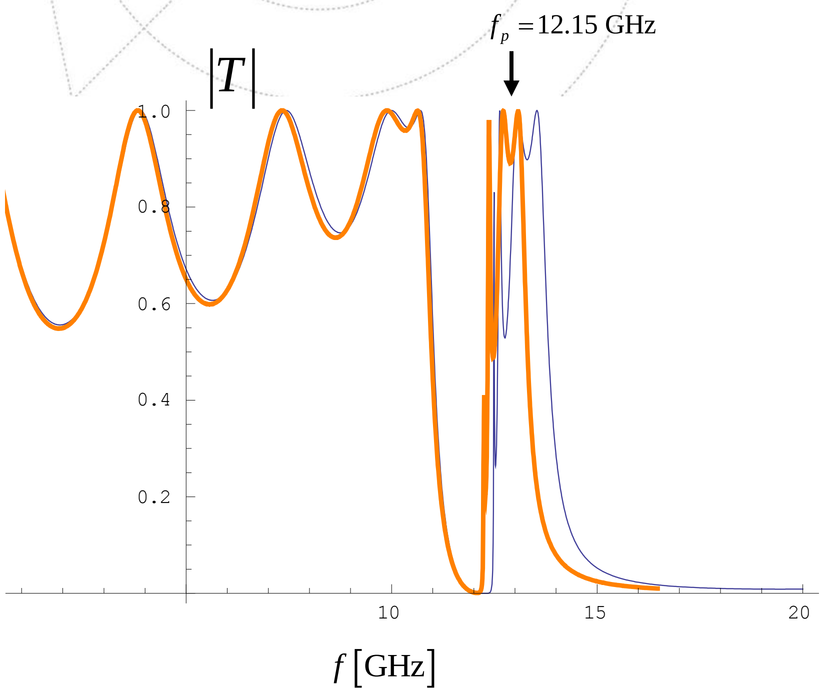
Numerical validation

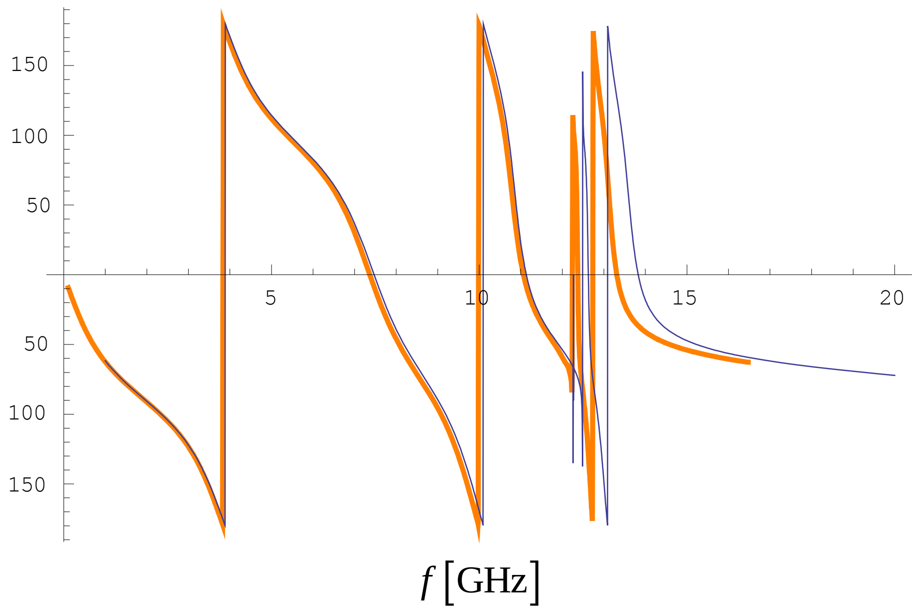


Analytical model

CSТ Microwave Studio

$$a = 2mm,\quad \varepsilon_h = 10.2,\quad g = 0.1a,\quad h = a,\quad r = 0.025a,\quad \theta_i = 45[\text{deg}],\ N_{\text{plates}} = 2+3$$





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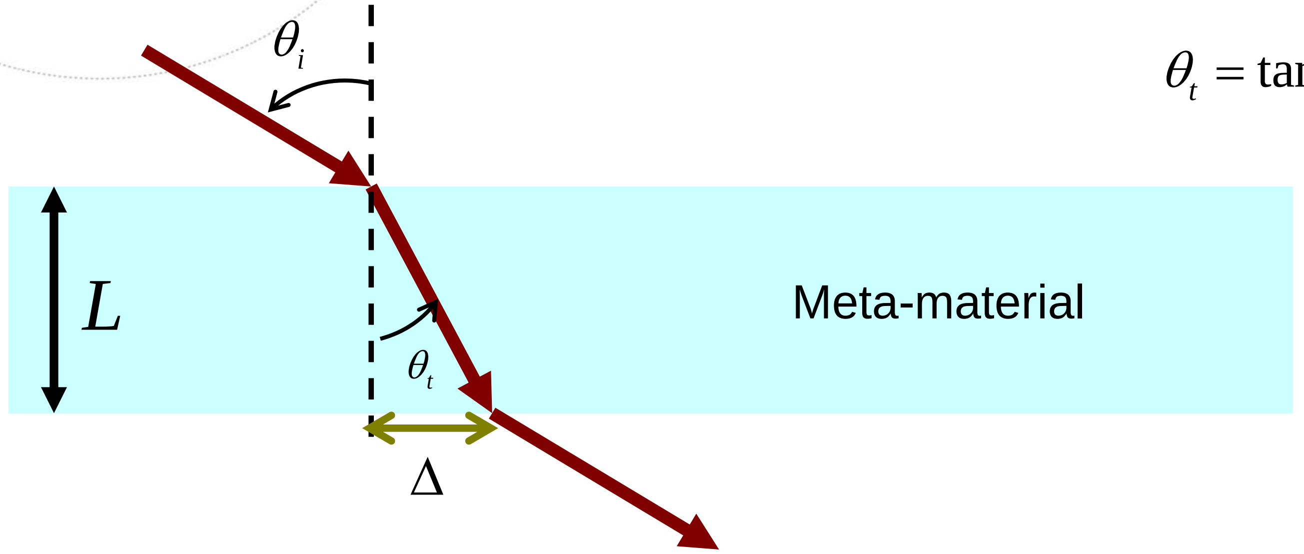
Characterization of negative refraction

- **Extraction of equivalent parameters from scattering data????**

There is a simpler and more general solution!



$$\theta_t = \tan^{-1}\frac{\Delta}{L}$$

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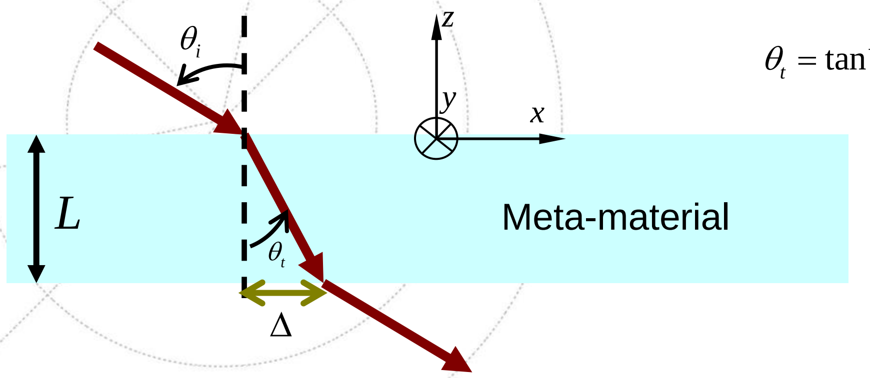
Characterization of negative refraction (contd.)



$$\theta_t = \tan^{-1}\frac{\Delta}{L}$$

Incident field (plane wave decomposition)**:**

$$H_y^i(x, z=0) = \int_{-\infty}^{+\infty} \tilde{H}_y^{inc}(k_x) e^{-jk_x x} dx$$

Transmitted wave:

$$H_y^o(x, z=-L) = \int_{-\infty}^{+\infty} \underbrace{T(\omega, k_x)}_{} \tilde{H}_y^{inc}(k_x) e^{-jk_x x} dx$$

Transfer function of the slab

(for plane wave incidence)

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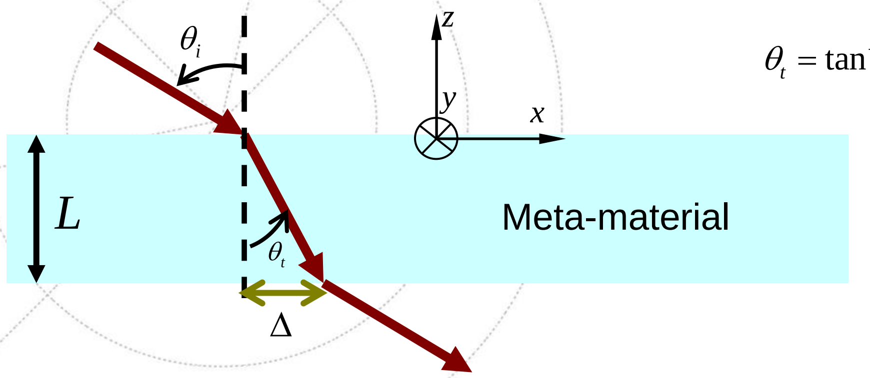
Characterization of negative refraction (concl.)



$$\theta_t = \tan^{-1}\frac{\Delta}{L}$$

$$H_y^o(x, z=-L) = \int_{-\infty}^{+\infty} T(\omega, k_x)\tilde{H}_y^{inc}(k_x)e^{-jk_x x}dx$$

Quasi-plane wave: $\tilde{H}_y^{inc}(k_x) \approx 0, \text{ except for } k_x = k_x^0 = \omega/c\sin\theta_i$

$$T(\omega, k_x) = |T|e^{j\phi} \quad \Rightarrow \quad T(\omega, k_x) \approx |T|_0 \, e^{j\left(\phi_0 + \frac{d\phi}{dk_x}\left(k_x - k_x^0\right)\right)}$$

$$H_y^o(x, z=-L) \approx T\left(\omega, k_x^0\right)e^{-j\Delta k_x^0} H_y^{inc}(x-\Delta)$$

$$\Delta = \frac{d\phi}{dk_x}$$

Negative refraction occurs when the phase of transfer function decreases with the incident angle!

M. G. Silveirinha, PHYSICAL REVIEW B 79, 153109, 2009

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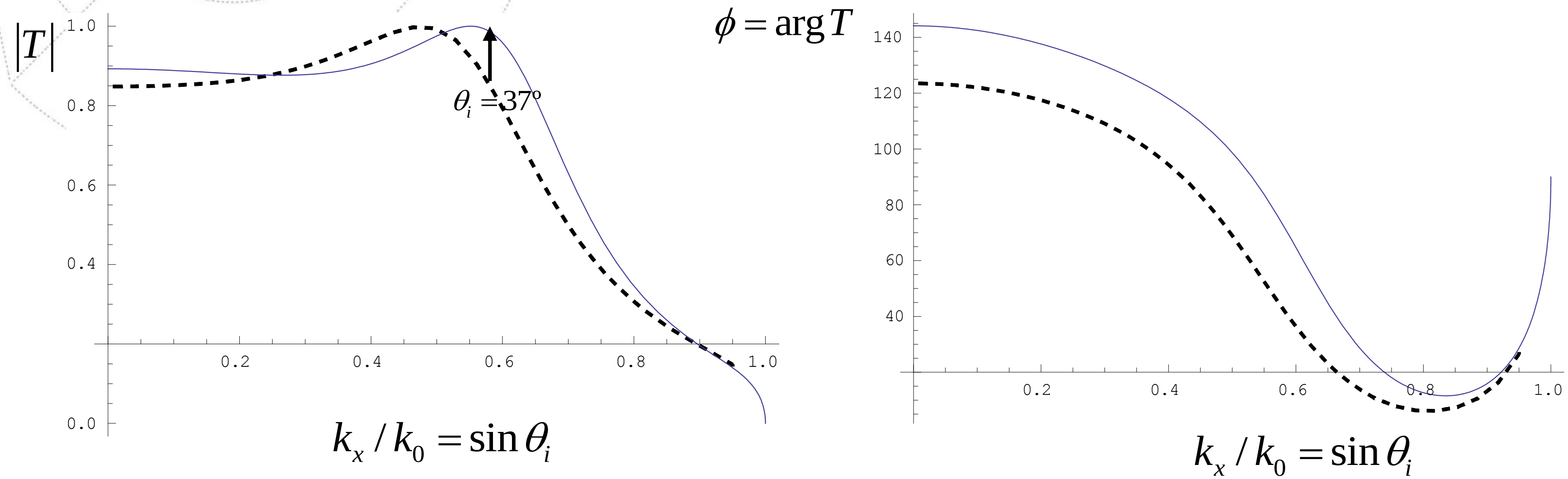
Negative Refraction in the Mushroom structure

$$a = 2mm, \quad \varepsilon_h = 10.2, \quad g = 0.1a, \quad h = a, \quad r = 0.025a, \quad N_{\text{plates}} = 2+3$$

Analytical model

CST Microwave Studio

$$f_0 = 11\text{GHz} \quad (\omega a / c = 0.46; \varepsilon_{zz} = -2.3)$$



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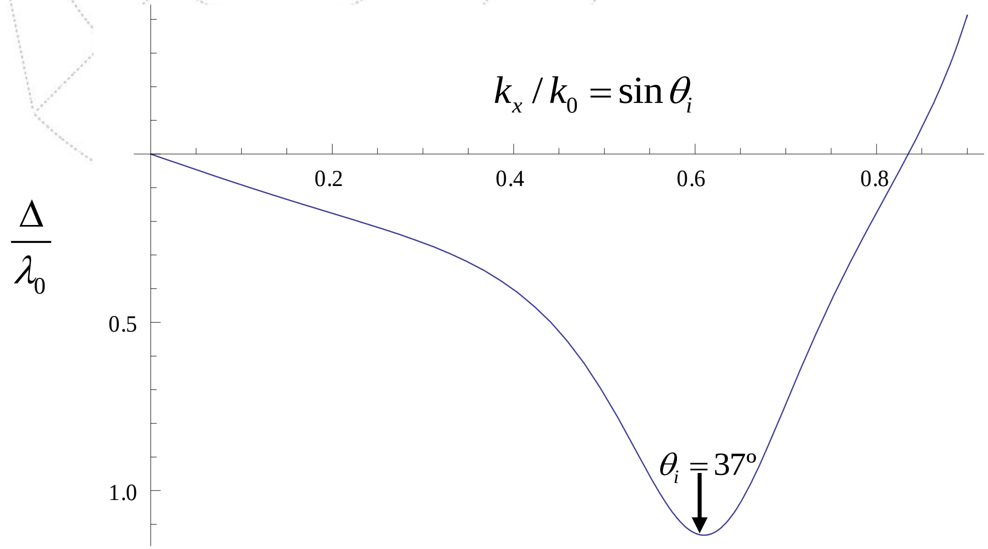
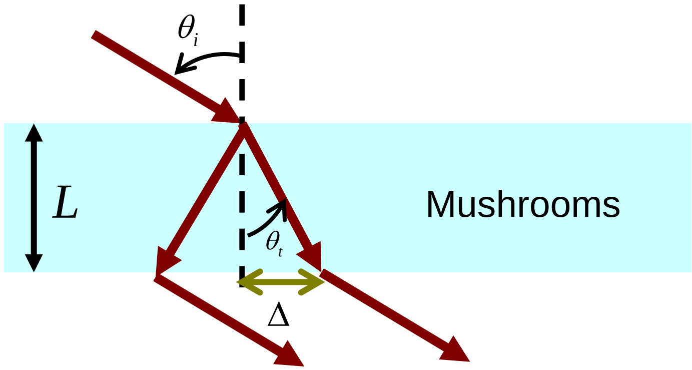
Negative spatial shift

$$a = 2mm, \quad \varepsilon_h = 10.2, \quad g = 0.1a, \quad h = a, \quad r = 0.025a, \quad N_{\text{plates}} = 2+3$$

$$f_0 = 11\text{GHz} \quad (\omega a / c = 0.46; \varepsilon_{zz} = -2.3)$$



$$\Delta = \frac{d\phi}{dk_x} < 0$$



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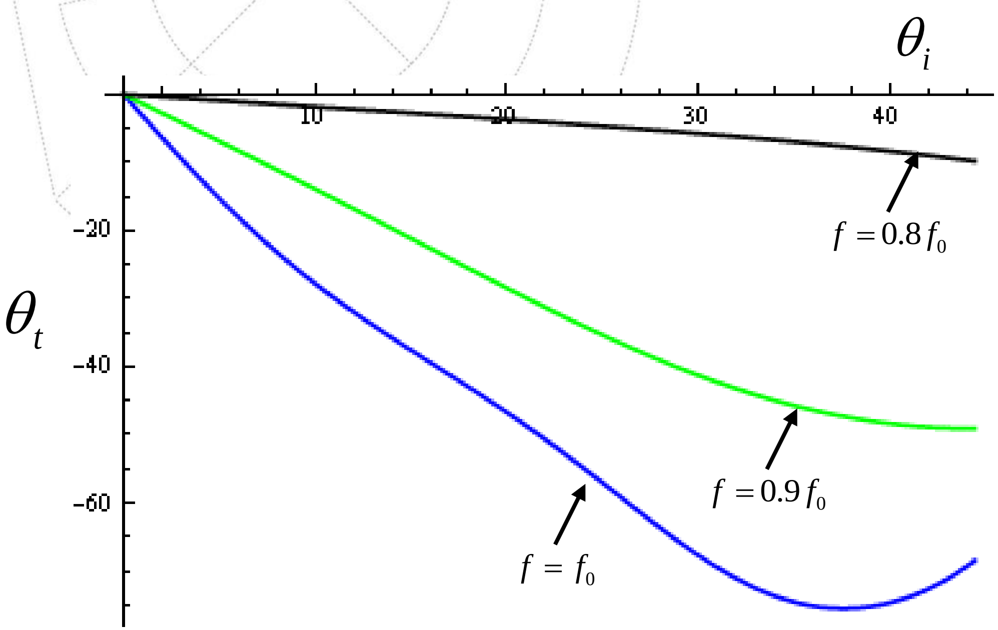
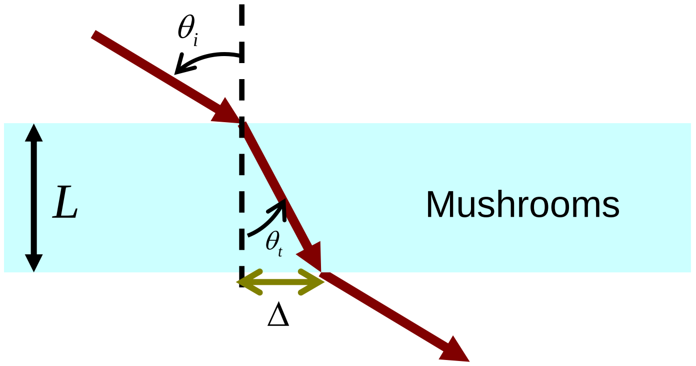
Negative angle of transmission

$$a = 2mm,\quad \varepsilon_h = 10.2,\quad g = 0.1a,\quad h = a,\quad r = 0.025a,\quad N_{\text{plates}} = 2+3$$

$$f_0 = 11\text{GHz} \quad (\omega a / c = 0.46; \varepsilon_{zz} = -2.3)$$





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Dependence with the number of layers

$$a = 2mm, \quad \varepsilon_h = 10.2, \quad g = 0.1a, \quad h = a, \quad r = 0.025a$$

Nplates	Δ	L	θ_i	θ_t
2+0	$-0.22\lambda_0$	$0.074\lambda_0$	22.96	-71.4
2+1	$-0.45\lambda_0$	$0.148\lambda_0$	29.09	-71.9
2+2	$-0.7\lambda_0$	$0.22\lambda_0$	31.33	-72.55
2+3	$-1\lambda_0$	$0.29\lambda_0$	32.73	-73.8
2+4	$-1.30\lambda_0$	$0.364\lambda_0$	34.68	-74.62
2+5	$-1.73\lambda_0$	$0.438\lambda_0$	35.76	-75.79

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Full wave simulations with CST Microwave Studio (incoming beam)

I will add here a slide with the incoming beam (without the slab)

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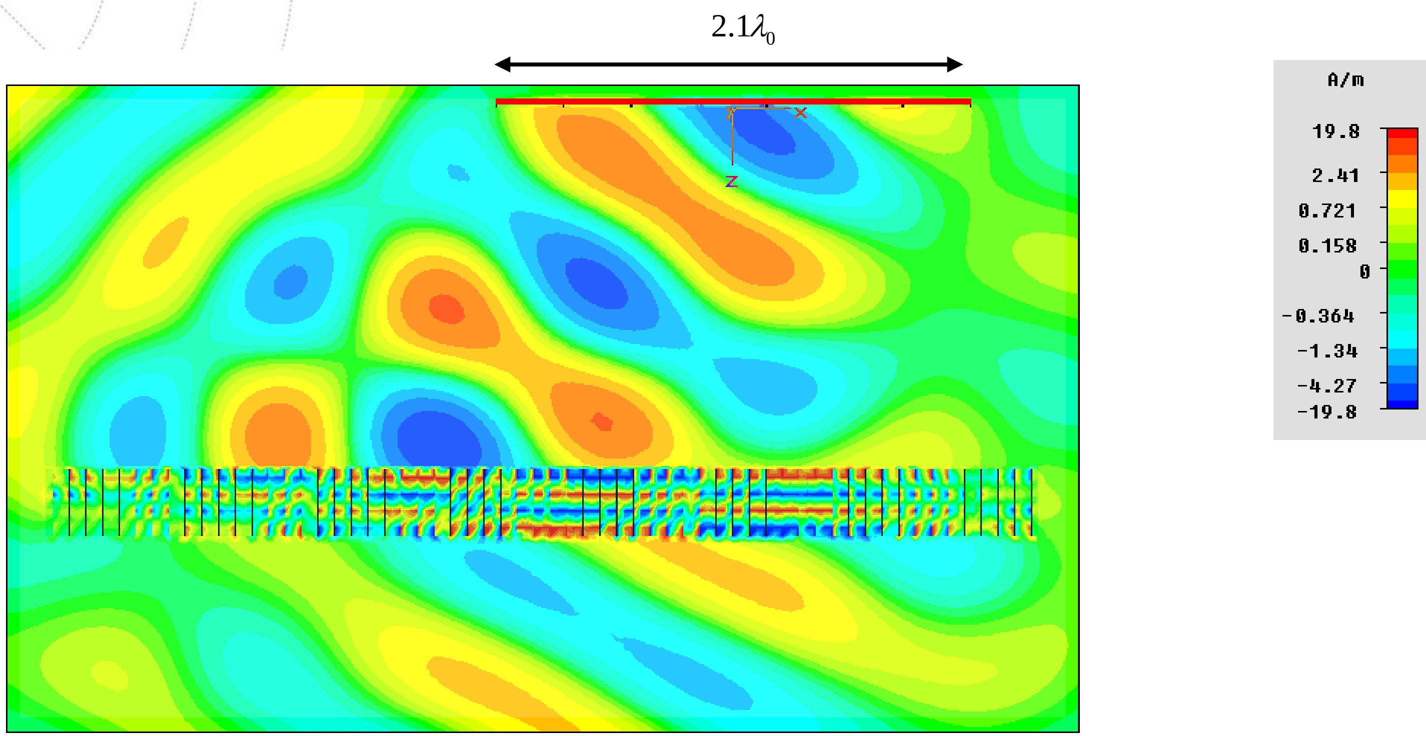
Full wave simulations with CST Microwave Studio (contd.)

$$a = 2mm, \quad \varepsilon_h = 10.2, \quad g = 0.1a, \quad h = a, \quad r = 0.025a, \quad N_{\text{plates}} = 2+3$$

$$f_0 = 11\text{GHz}, \ \theta_i = 33^{\circ}$$

H_y-component



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Conclusion

- **A multilayered mushroom structure behaves as a local uniaxial ENG material periodically loaded with patch grids**

- **Strong negative refraction may occur at an interface with air at microwave frequencies**

- **This phenomenon can be accurately described using homogenization techniques.**

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