High-Impedance Surfaces with Graphene Patches as Absorbing Structures at Microwaves

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Third International Congress on Advanced Electromagnetic Materials in Microwaves and Optics
London, United Kingdom
30 August – 4 September, 2009
Homogenization models for the analysis of high-impedance surfaces with graphene (two-dimensional semi-metal) patches with and without vias

- Dynamic model for HIS with graphene patches (no vias)
  - grid impedance of graphene patches
  - circuit theory model

- Non-local model for mushroom-type HIS with graphene patches
  - Additional Boundary Condition (ABC)
  - spatial dispersion of wire-medium slab
Graphene

- **Graphene** is a mono-atomic layer of graphite
- A single-wall carbon nanotube is a rolled-up sheet of graphene
- Although graphene has been long studied to explain the properties of carbon systems, it was long thought that graphene itself did not exist

2004 – graphene found!
Graphene is moderately easy to make, and is visible in an optical microscope when residing on oxidized Si with a certain SiO₂ thickness due to a weak interference effect.
Graphene – New Generation of Transistors

Moving towards a graphene world

Welcome to graphene; the flat carbon sheet with revolutionary aspirations. This thinnest possible pencil-lead shaving has already interested theoretical physicists with its electronic properties, and is predicted to edge aside silicon in the microchips of the future. Now it’s ready for its first practical application.

Electronic properties make graphene a candidate to replace silicon in a fresh era of microchip electronics. “Graphene is quite different from conventional semiconductors such as silicon,” explains Philip Kim, a physicist from Columbia University in New York. Electrons move though silicon in a series of collisions; these

Graphene can be gated, and has long spin-coherence length and high mobility at room temperature

μ greater than 15,000 cm²/Vs have been measured, and 200,000 cm²/Vs are predicted to be possible
Surface Conductivity of Graphene

- No magnetic bias field
- Spatial dispersion – not important at microwaves

\[
\sigma = -j \frac{e^2 k_B T}{\pi \hbar^2 (\omega - j 2 \Gamma)} \left( \frac{\mu_c}{k_B T} + 2 \ln \left( e^{-\mu_c / k_B T} + 1 \right) \right)
\]

-\(e\): charge of an electron
-\(k_B\): Boltzmann’s constant
-\(\mu_c\): chemical potential (electrostatic bias)
-\(\Gamma\): electron scattering rate (10^{12} Hz)
-\(T\): temperature (300 K)

JOURNAL OF APPLIED PHYSICS 103, 064302 (2008)

Dyadic Green’s functions and guided surface waves for a surface conductivity model of graphene

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Plane-Wave Incidence
Analytical Modeling of Graphene HIS Structures

- Dynamic solution of 2D strip grid scattering problem
- Averaged impedance boundary condition
- Approximate Babinet principle

Transmission-line network

\[ Z_s = \frac{Z_g Z_d}{Z_g + Z_d} \]

Graphene patches

Simple and Accurate Analytical Model of Planar Grids and High-Impedance Surfaces Comprising Metal Strips or Patches

Olli Luukkonen, Constantin Simovski, Member, IEEE, Gérard Granet, George Goussetis, Member, IEEE, Dmitri Lisoubichenko, Antti V. Räisänen, Fellow, IEEE, and Sergei A. Tretyakov, Fellow, IEEE
Grid Impedance of Graphene Patches and Strips

\[
Z_{g_{TE}}^{TE} = \frac{p}{(p-g)\sigma} - j\frac{\eta_{\text{eff}}}{2\alpha} \left(1 - \frac{k_0^2}{2k_{\text{eff}}^2} \sin^2 \theta \right)
\]

\[
Z_{g_{TM}}^{TM} = \frac{p}{(p-g)\sigma} - j\frac{\eta_{\text{eff}}}{2\alpha} \left(1 - \frac{k_0^2}{k_{\text{eff}}^2} \sin^2 \theta \right)
\]

\[
Z_{g_{TM}}^{TM} = \frac{p}{(p-w)\sigma} + j\frac{\eta_{\text{eff}}}{2\alpha} \left(1 - \frac{k_0^2}{k_{\text{eff}}^2} \sin^2 \theta \right)
\]

\[
Z_{g_{TE}}^{TE} = \frac{p}{(p-g)\sigma} + j\frac{\eta_{\text{eff}}}{2\alpha} \left(1 - \frac{k_0^2}{k_{\text{eff}}^2} \sin^2 \theta \right)
\]

\[
\alpha = \frac{k_{\text{eff}}p}{\pi} \ln \left(\csc \left(\frac{\pi g}{2p}\right)\right)
\]

\[
\alpha = \frac{k_{\text{eff}}p}{\pi} \ln \left(\csc \left(\frac{\pi w}{2p}\right)\right)
\]
Surface Impedance of Grounded Slab

**Dielectric Impedance**

**TM**

\[ Z_{d}^{TM}(\omega, \theta) = \frac{j \eta_0}{\sqrt{\varepsilon_r - \sin^2 \theta}} \tan(k_{zd} h) \left(1 - \frac{\sin^2 \theta}{\varepsilon_r}\right) \]

**TE**

\[ Z_{d}^{TE}(\omega, \theta) = \frac{j \eta_0}{\sqrt{\varepsilon_r - \sin^2 \theta}} \tan(k_{zd} h) \]

**vertical component of the wave vector of the refracted wave**

\[ k_{zd} = \omega \sqrt{\varepsilon_0 \mu_0} \sqrt{\varepsilon_r - \sin^2 \theta} \]
Graphene HIS at Oblique Incidence

D = 2 mm, g = 0.2 mm, h = 1 mm
\( \varepsilon_r = 10.2 \)

TM polarization

Reflection properties are sensitive to the angle of incidence

\( \mu_c = 0.5185 \text{ eV} \)
Tunable Graphene HIS

D = 2 mm, g = 0.2 mm, h = 1 mm
\( \varepsilon_r = 10.2 \)

**TM polarization**

Reflection minima obtained at different incident angles by adjusting the chemical potential

Solid lines – analytical model
Dashed lines – FEM results

Tunable Graphene HIS

D = 2 mm, g = 0.2 mm, h = 1 mm
\( \varepsilon_r = 10.2 \)

**TE polarization**

Reflection minima at different incident angles are obtained in a narrow frequency range by adjusting the chemical potential

Solid lines – analytical model
HIS with PEC Patches and Lossy Dielectric Slab

\[ \varepsilon_r = 10.2(1 - j\times0.26) \]

D = 2 mm, g = 0.2 mm, h = 1 mm

TM polarization

Reflection minima are sensitive to the angle of incidence

Solid lines – analytical model
Mushroom Array with Graphene Patches

Non-Local Model – SD + ABC

Wire medium slab as anisotropic material characterized by effective permittivity

Spatial dispersion

\[
\tilde{\varepsilon}_{\text{eff}} = \varepsilon_0 \varepsilon_r \left( \hat{x}\hat{x} + \varepsilon_{yy} \hat{y}\hat{y} + \hat{z}\hat{z} \right)
\]

\[
\varepsilon_{yy} = 1 - \frac{\beta_p^2}{\beta_r^2 - k_y^2}
\]

\[
\beta_r = \beta \sqrt{\varepsilon_r}
\]

\[
\beta \approx \omega / c
\]

\[
\beta_p^2 = \frac{2\pi / a^2}{\ln \left( \frac{a}{2\pi r_0} \right)} + 0.5275
\]

SD + ABC Model

TM-polarized incident plane wave excites TEM and TM modes in the wire medium slab

\[
H_x = \left( e^{+\gamma_0 y} + \rho e^{-\gamma_0 y} \right) e^{-jkz} \quad \text{air region}
\]

\[
H_x = \left( B_{TEM} \cos(\beta_r y) + B_{TM} \cosh(\gamma_{TM} y) \right) e^{-jkz} \quad \text{wire medium slab}
\]

- Two-sided impedance boundary condition at \( y=L \):

\[
E_z \mid_{y=L^-} = E_z \mid_{y=L^+} = -Z_g \left( H_x \mid_{y=L^+} - H_x \mid_{y=L^-} \right)
\]

- Additional boundary condition at the via-graphene patch connection at \( y=L \):

\[
\left[ \frac{\sigma}{j\omega \varepsilon_0 \varepsilon_r} \frac{dI(y)}{dy} + I(y) \right] \mid_{y=L^-} = 0
\]

In terms of field components:

\[
\left[ \frac{\sigma}{j\omega \varepsilon_0 \varepsilon_r} \left( \beta \varepsilon_r \frac{dE_y}{dy} + k_z \eta_0 \frac{dH_x}{dy} \right) + \left( \beta \varepsilon_r E_y + k_z \eta_0 H_x \right) \right] \mid_{y=L^-} = 0
\]

- Additional boundary condition at the via-ground plane connection at \( y=0 \):

\[
\left. \frac{dI(y)}{dy} \right|_{y=0^+} = 0
\]

In terms of field components:

\[
\left. \beta \varepsilon_r \frac{dE_y}{dy} + k_z \eta_0 \frac{dH_x}{dy} \right|_{y=0^+} = 0
\]
Reflection Coefficient

- Reflection coefficient

\[
\rho = \frac{\coth(\gamma_{TM}L) \cot(\beta_r L) \times K - \left(\frac{1}{\gamma_0} + j \frac{\eta_0}{Z_g k_0}\right)}{\coth(\gamma_{TM}L) \cot(\beta_r L) \times K + \left(\frac{1}{\gamma_0} - j \frac{\eta_0}{Z_g k_0}\right)}
\]

where

\[
K = \frac{\left(\frac{1}{\varepsilon_{yy}^{TM}} - 1\right)\left(\frac{\sigma}{j\omega\varepsilon_0\varepsilon_r} \tanh(\gamma_{TM}L) + 1\right) + \left(1 - \frac{\sigma\beta_r}{j\omega\varepsilon_0\varepsilon_r} \tan(\beta_r L)\right)}{-\frac{\beta_r}{\varepsilon_r} \left(\frac{1}{\varepsilon_{yy}^{TM}} - 1\right)\left(\frac{\sigma\gamma_{TM}}{j\omega\varepsilon_0\varepsilon_r} + \coth(\gamma_{TM}L)\right) + \frac{\gamma_{TM}}{\varepsilon_r} \left(\cot(\beta_r L) - \frac{\sigma\beta_r}{j\omega\varepsilon_0\varepsilon_r}\right)}
\]

\[
\varepsilon_{yy}^{TM} = 1 - \frac{\beta_p^2}{k_z^2 + \beta_p^2}
\]

\[
\gamma_{TM} = \sqrt{\beta_p^2 + k_z^2 - \beta_r^2}
\]

\[
\gamma_0 = \sqrt{k_z^2 - \beta^2}
\]

In the limiting case \(\sigma \to 0\) (transparent patches) it turns to wire-medium slab:


In the limiting case \(\sigma \to \infty\) (PEC patches) it turns to mushroom HIS:

**Luukkonen et al., IEEE Trans. Microwave Theory Tech., 2009 (to appear)**

**Yakovlev et al., IEEE Trans. Microwave Theory Tech., 2009 (to appear)**
Mushroom Array with Graphene Patches

Period: 2 mm
Gap: 0.2 mm
Radius of vias: 0.05 mm
Substrate thickness: 1 mm
Dielectric permittivity: 10.2
Mushroom HIS with graphene patches results in stable resonance frequencies (in the vicinity of the plasma frequency) for different angles of incidence.
Tunable Mushroom HIS with Graphene Patches

**TM polarization**

In mushroom HIS with graphene patches the reflection minima can be obtained at different incident angles by **adjusting the chemical potential**
Mushroom HIS with PEC Patches and Lossy Dielectric Slab

Comparison with HIS without vias

TM polarization

Mushroom HIS

Patch array HIS (no vias)

Mushroom HIS results in better absorption, however, in both cases the reflection coefficient is sensitive to the angle of incidence.
Microscopic Current Along the Vias

For large chemical potential [electrostatic bias field] the microscopic current along the vias is close to uniform, and the spatial dispersion effects in wire medium are significantly reduced.
Conclusions

- **Dynamic** model for HIS with graphene patches and **non-local (SD+ABC)** model for mushroom HIS with graphene patches are proposed for the analysis of absorption properties at microwaves.

- The reflection minima in HIS structures with graphene patches (with and without vias) can be obtained at different incident angles by **adjusting the chemical potential** (electrostatic bias field).

- For large values of chemical potential the microscopic current along the vias is close to uniform, and the **spatial dispersion** effects in wire-medium are significantly reduced.
Acknowledgement

The authors are thankful to Mário G. Silveirinha for helpful discussions regarding the Additional Boundary Condition