

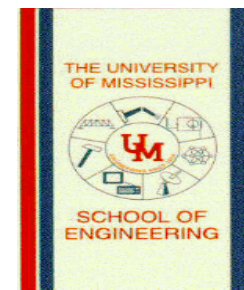


Green's Functions for High-Impedance Surfaces: A Comparison between Homogenized Models and Full-Wave Results

P. Baccarelli⁽¹⁾, P. Burghignoli⁽¹⁾, G. W. Hanson⁽²⁾,
G. Lovat⁽¹⁾, S. Paulotto^{(1),(3)}, and A. B. Yakovlev⁽⁴⁾



SAPIENZA
UNIVERSITÀ DI ROMA



⁽¹⁾"La Sapienza" University of Rome ⁽²⁾University of Wisconsin ⁽³⁾University of Houston ⁽⁴⁾University of Mississippi

Outline

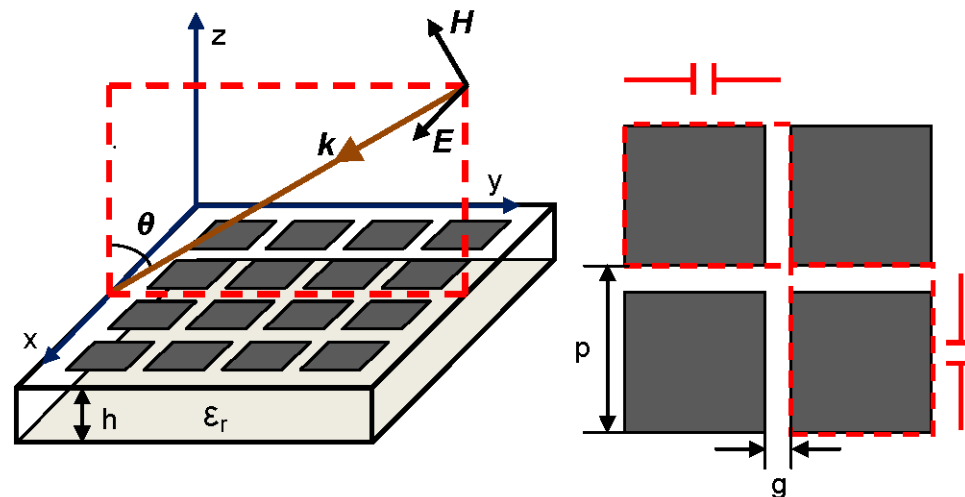
- **Motivation and Background**
 - High Impedance Surfaces and Artificial Magnetic Conductors
 - Plane-Wave Incidence
- **Excitation with an Electric Line Source**
 - Homogenized Model:
 - Green's function
 - Line source vs. plane-wave incidence
 - Periodic MoM Analysis: Array Scanning Method
- **Numerical Results**
- **Conclusions**

Motivation and Background

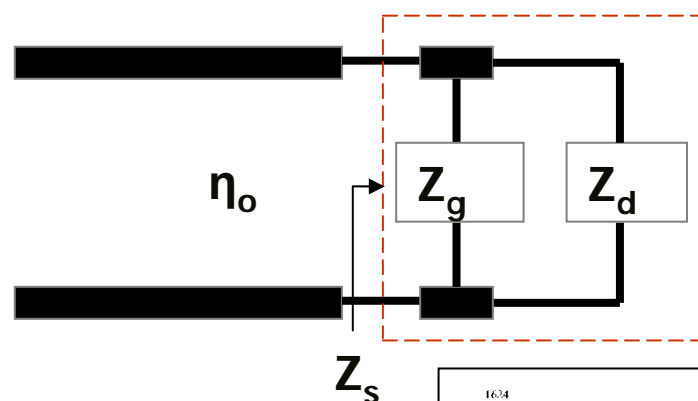
Plane-Wave Incidence

Analytical Modeling of HIS Structures

- Dynamic solution of 2D strip grid scattering problem
- Averaged impedance boundary condition
- Approximate Babinet principle



Transmission-line network



HIS

$$Z_s = \frac{Z_g Z_d}{Z_g + Z_d}$$

Parallel resonance

$$Z_g(\omega) + Z_d(\omega) = 0$$

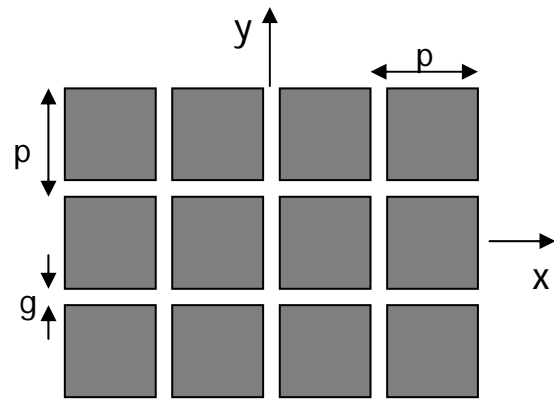
1634

IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, VOL. 56, NO. 6, JUNE 2008

Simple and Accurate Analytical Model of Planar
Grids and High-Impedance Surfaces Comprising
Metal Strips or Patches

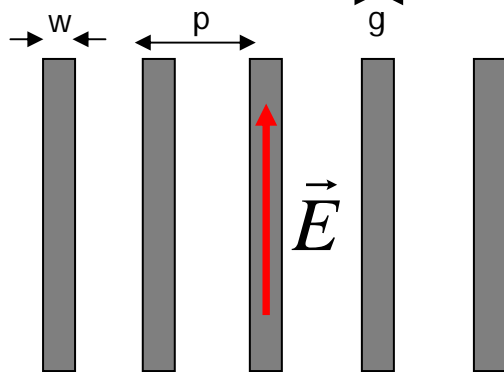
Olli Luukkonen, Constantin Simovski, Member, IEEE, Gérard Granet, George Goussetis, Member, IEEE,
Dmitri Lioubtchenko, Antti V. Räsänen, Fellow, IEEE, and Sergei A. Tretyakov, Fellow, IEEE

Grid Impedance



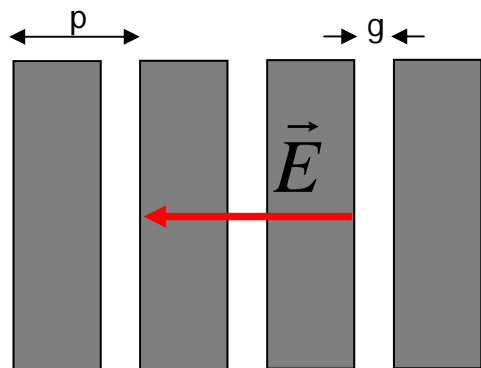
$$Z_g^{TE} = -j \frac{\eta_{eff}}{2\alpha} \frac{1}{\left(1 - \frac{1}{2} \frac{k_0^2}{k_{eff}^2} \sin^2 \theta\right)}$$

$$Z_g^{TM} = -j \frac{\eta_{eff}}{2\alpha} \quad \alpha = \frac{k_{eff} p}{\pi} \ln \left(\csc \left(\frac{\pi g}{2p} \right) \right)$$



$$Z_g^{TM} = j \frac{\eta_{eff}}{2} \alpha \left(1 - \frac{k_0^2}{k_{eff}^2} \sin^2 \theta\right)$$

$$Z_g^{TE} = j \frac{\eta_{eff}}{2} \alpha \quad \alpha = \frac{k_{eff} p}{\pi} \ln \left(\csc \left(\frac{\pi w}{2p} \right) \right)$$



$$Z_g^{TE} = -j \frac{\eta_{eff}}{2\alpha} \frac{1}{\left(1 - \frac{k_0^2}{k_{eff}^2} \sin^2 \theta\right)}$$

$$Z_g^{TM} = -j \frac{\eta_{eff}}{2\alpha}$$

Surface Impedance of Grounded Slab

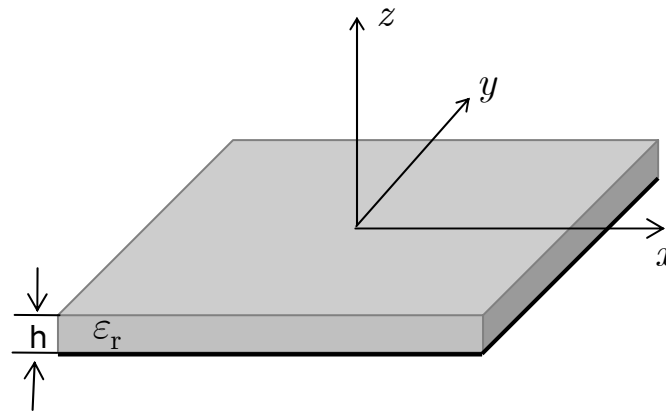
Dielectric Impedance

$$\text{TM} \quad Z_d^{TM}(\omega, \theta) = \frac{j\eta_0}{\sqrt{\epsilon_r - \sin^2 \theta}} \tan(k_{zd} h) \left(1 - \frac{\sin^2 \theta}{\epsilon_r} \right)$$

$$\text{TE} \quad Z_d^{TE}(\omega, \theta) = \frac{j\eta_0}{\sqrt{\epsilon_r - \sin^2 \theta}} \tan(k_{zd} h)$$

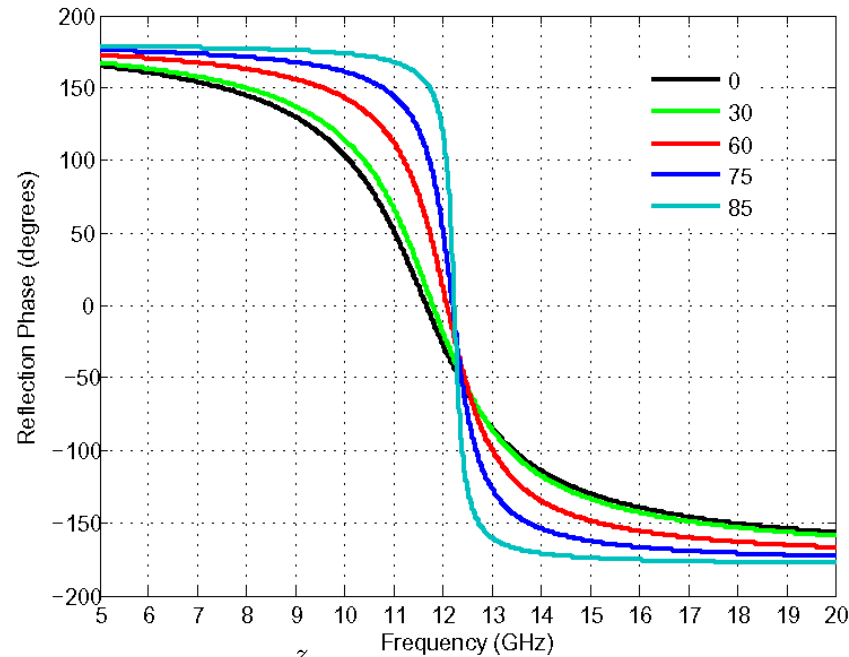
$$k_{zd} = \omega \sqrt{\epsilon_0 \mu_0} \sqrt{\epsilon_r - \sin^2 \theta}$$

vertical component of the
wave vector of the refracted wave

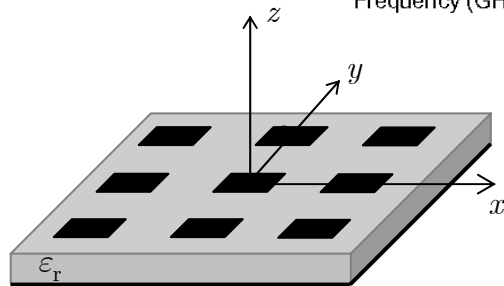
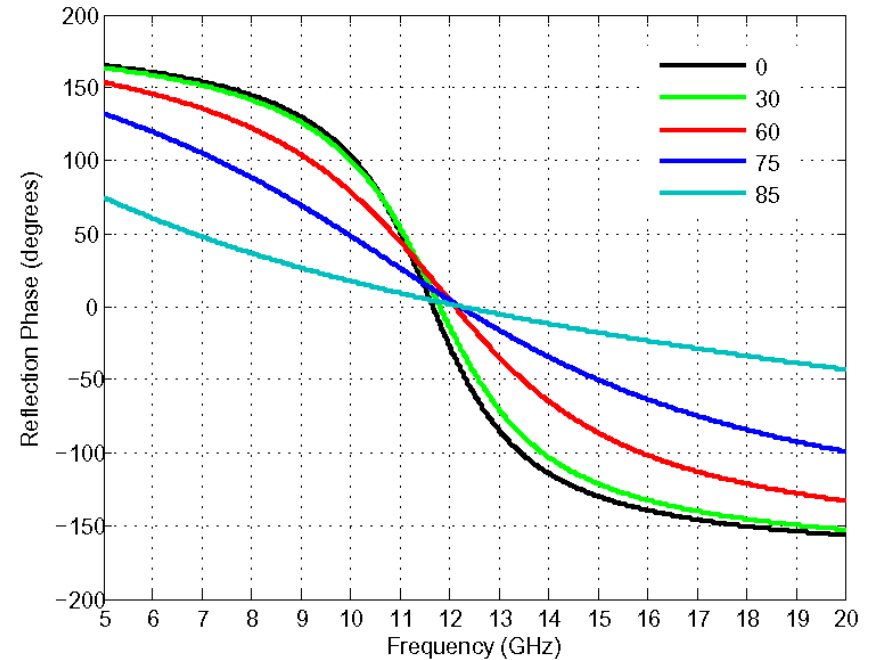


Patch Array HIS – Oblique Incidence

TE polarization



TM polarization



Substrate thickness: 1 mm
Period: 2 mm
Gap: 0.2 mm
Dielectric permittivity: 10.2

1634

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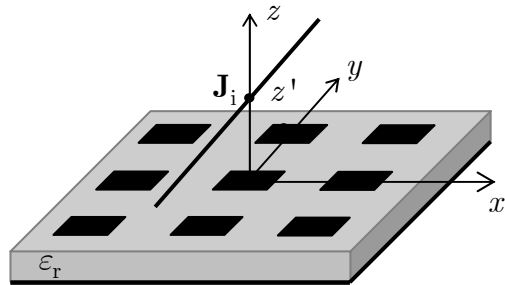
Simple and Accurate Analytical Model of Planar Grids and High-Impedance Surfaces Comprising Metal Strips or Patches

Olli Luukkonen, Constantin Simovski, *Member, IEEE*, Gérard Granet, George Goussetis, *Member, IEEE*, Dmitri Lioubtchenko, Antti V. Räisänen, *Fellow, IEEE*, and Sergei A. Tretyakov, *Fellow, IEEE*

Excitation with a Line Source

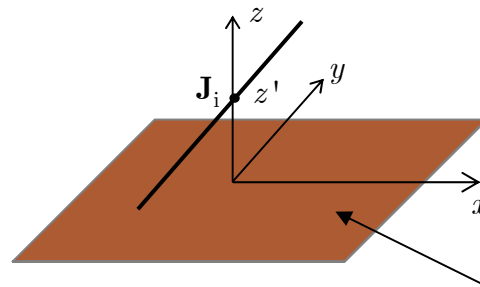
Green's Function for Homogenized HIS

Original Problem



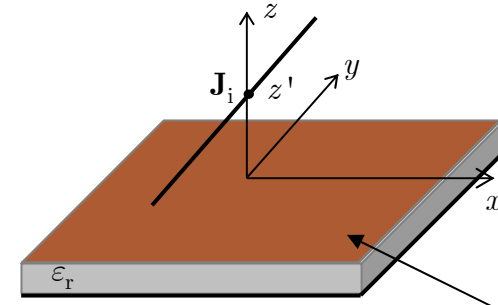
$$\mathbf{J} = \hat{\mathbf{y}} I_0 \delta(z - z_0) \delta(x)$$

Homogenized Problem
Method I



$$Z_s = Z_{\text{grid}} \parallel Z_{\text{dielectric}}$$

Homogenized Problem
Method II



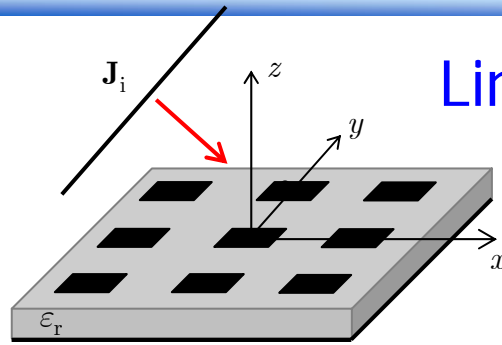
$$Z_{\text{grid}}$$

$$\mathbf{E}(x, z) = \hat{\mathbf{z}} \frac{I_0 k_1^2}{j\omega\epsilon_1} \left(\underbrace{\frac{1}{4j} H_0^{(2)} \left(k_1 \sqrt{(z - z_0)^2 + x^2} \right)}_{\text{Direct Term}} + \underbrace{\frac{1}{2\pi} \int_{-\infty}^{\infty} R_t(k_x) e^{-p_1(x+z_0)} \frac{e^{jk_x x}}{2\sqrt{k_x^2 - k_1^2}} dk_x}_{\text{Scattered Term}} \right)$$

$$R_t^{\text{method I}}(k_x) = \frac{p_1 - j\omega\mu_1/Z_s}{p_1 + j\omega\mu_1/Z_s} \quad p_1 = \sqrt{k_x^2 - k_1^2} \quad M^2 = \mu_2/\mu_1$$

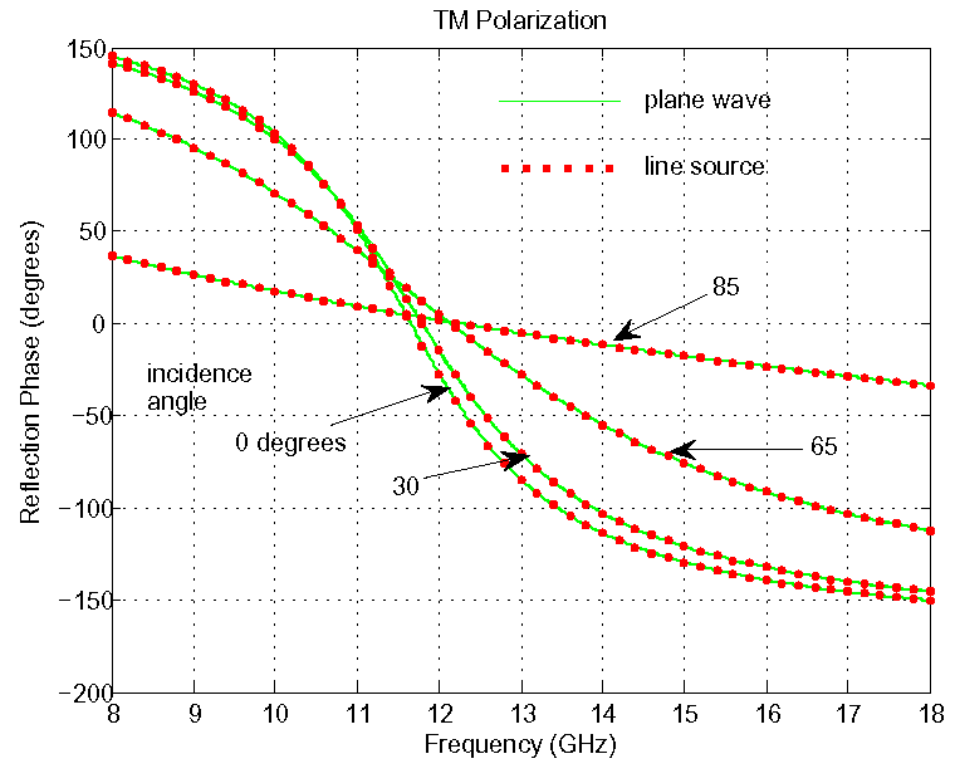
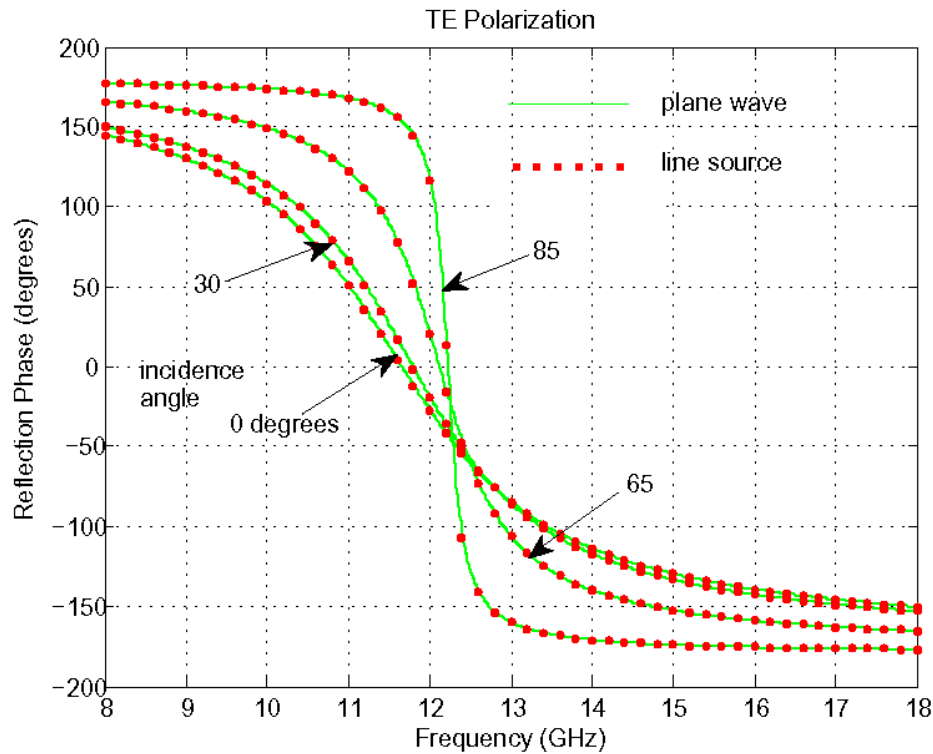
$$R_t^{\text{method II}}(k_x) = \frac{(1 - e^{2p_2 h})(p_1 - j\omega\mu_1/Z_g)M^2 + (1 + e^{2p_2 h})p_2}{(1 - e^{2p_2 h})(p_1 + j\omega\mu_1/Z_g)M^2 - (1 + e^{2p_2 h})p_2}$$

Line Source versus Plane-Wave Incidence (1)



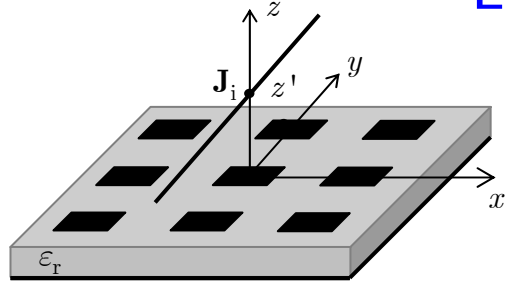
Line source in the far zone

Substrate thickness: 1 mm
Period: 2 mm
Gap: 0.2 mm
Dielectric permittivity: 10.2



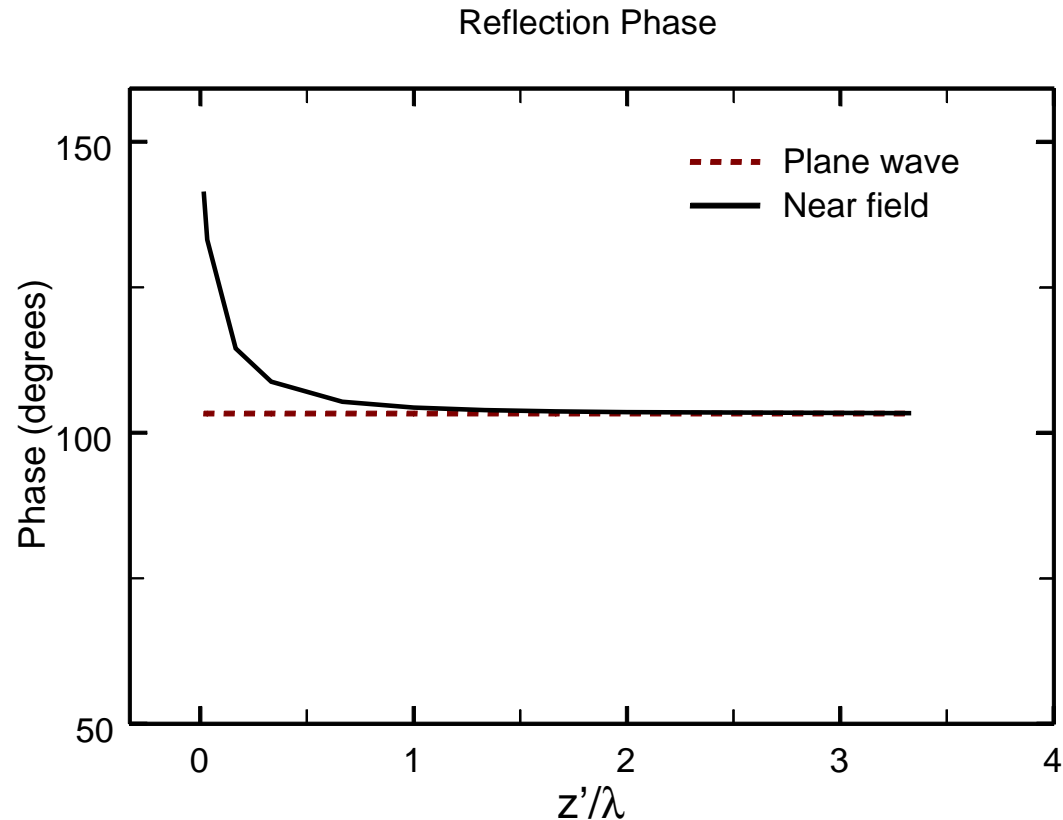
Line Source versus Plane-Wave Incidence (2)

Line source in proximity of homogenized HIS



$f = 10 \text{ GHz}$

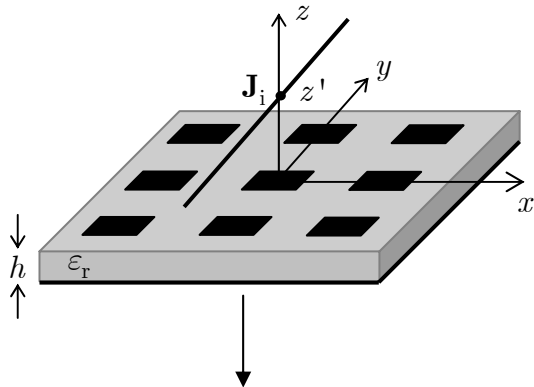
Normal incidence



The plane-wave and line-source results agree when the source is positioned about one wavelength above the HIS

Periodic Analysis: ASM 1D – 2D

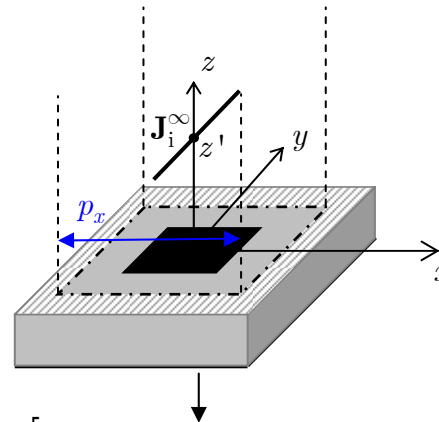
Original problem



$$\mathbf{J}_i(x, z) = \mathbf{y}_0 I_0 \delta(x) \delta(z - z')$$

One-dimensional (1D) excitation

Auxiliary Floquet-Periodic (FP) problem



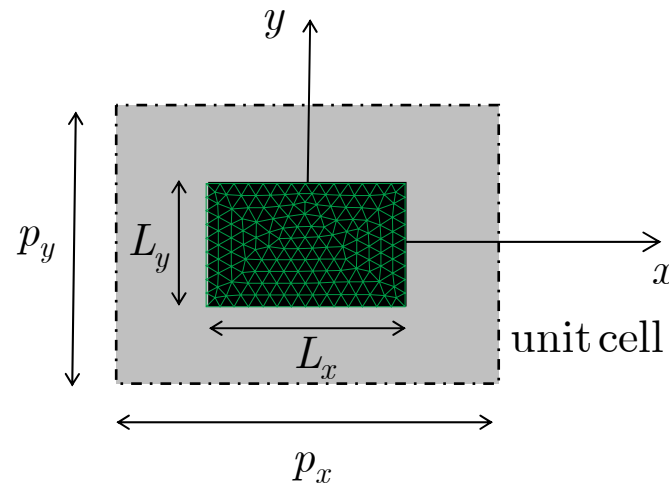
$$\mathbf{J}_i^\infty(x, z) = \mathbf{y}_0 I_0 \left[\sum_{m=-\infty}^{+\infty} \delta(x - mp_x) e^{-jmk_x p_x} \right] \delta(z - z')$$

Two-dimensional (2D) periodicity

The ASM requires the efficient solution of a large number of auxiliary FP sub-problems

Periodic Analysis: The Auxiliary FP Problem

Mixed
Potential
Integral
Equations
(MPIE)



Rao
Wilton
Glisson
(RWG)

Discretization
of arbitrary planar
geometry
within the Unit Cell

Special Issue on Metamaterials EBG

**Full-wave analysis of bound and leaky modes
propagating along 2D periodic printed structures
with arbitrary metallisation in the unit cell**

P. Baccarelli, S. Paulotto and C. Di Nallo

IET Microw. Antennas Propag., Vol. 1, No. 1, February 2007

Acceleration techniques and interpolation schemes are required for the computation of the Mixed-Potential Multilayered 2-D Periodic Green's Functions

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IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, VOL. 57, NO. 1, JANUARY 2009

Regularization of Mixed-Potential Layered-Media
Green's Functions for Efficient Interpolation
Procedures in Planar Periodic Structures

Guido Valerio, *Student Member, IEEE*, Paolo Baccarelli, *Member, IEEE*, Simone Paulotto, *Member, IEEE*,
Fabrizio Frezza, *Senior Member, IEEE*, and Alessandro Galli, *Member, IEEE*

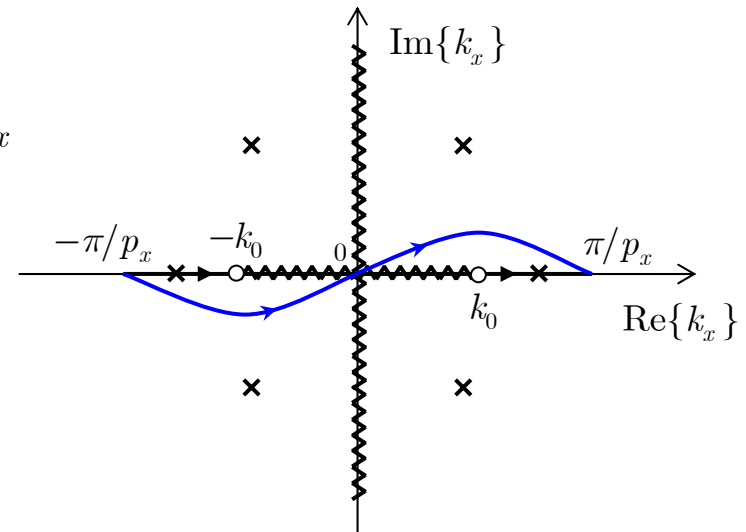
Periodic Analysis: ASM

$$\delta(x) = \frac{p_x}{2\pi} \int_{-\pi/p_x}^{+\pi/p_x} \sum_{m=-\infty}^{+\infty} \delta(x - mp_x) e^{-jm k_x p_x} dk_x$$

Basic ASM identity

→ $\mathbf{J}_i(x, z) = \delta(z - z') \frac{p_x}{2\pi} \int_{-\pi/p_x}^{+\pi/p_x} \mathbf{J}_i^\infty(x, z; k_x) dk_x$

→ $\mathbf{E}(x, y, z) = \frac{p_x}{2\pi} \int_{-\pi/p_x}^{+\pi/p_x} \mathbf{E}^\infty(x, y, z; k_x) dk_x$

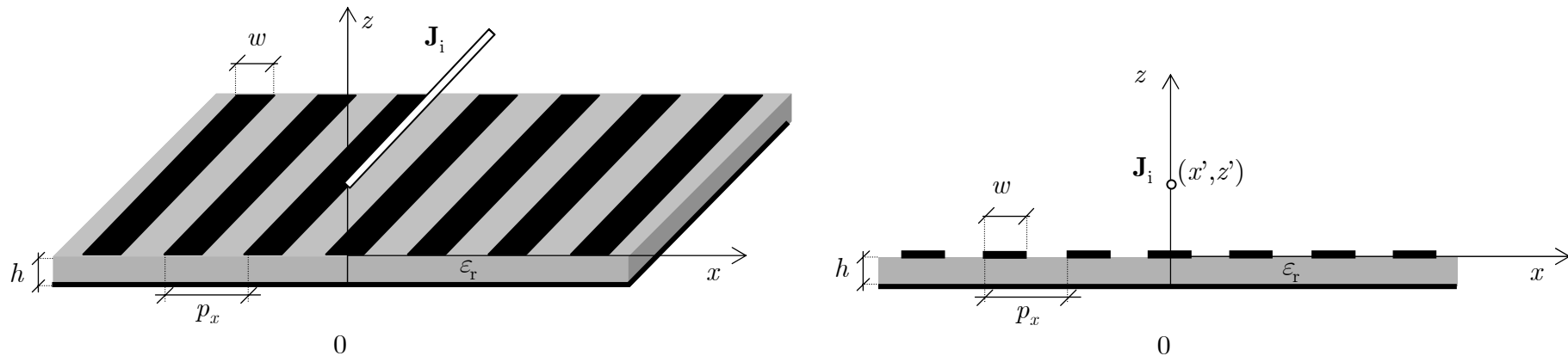


The field is represented as an integral superposition of FP fields. The integration is performed with respect to the phase shift k_x in the direction x over the entire first Brillouin zone.

Numerical Results

Comparison between Homogenized Model and Full-Wave Results

Validation: MSG-GDS



In this case both the structure and the source are independent of the y direction. The excited field is also independent of y and is purely TE_{xz} .

A one-dimensional (1D) ASM with 1D periodicity is implemented ([ASM 1D-1D](#)).

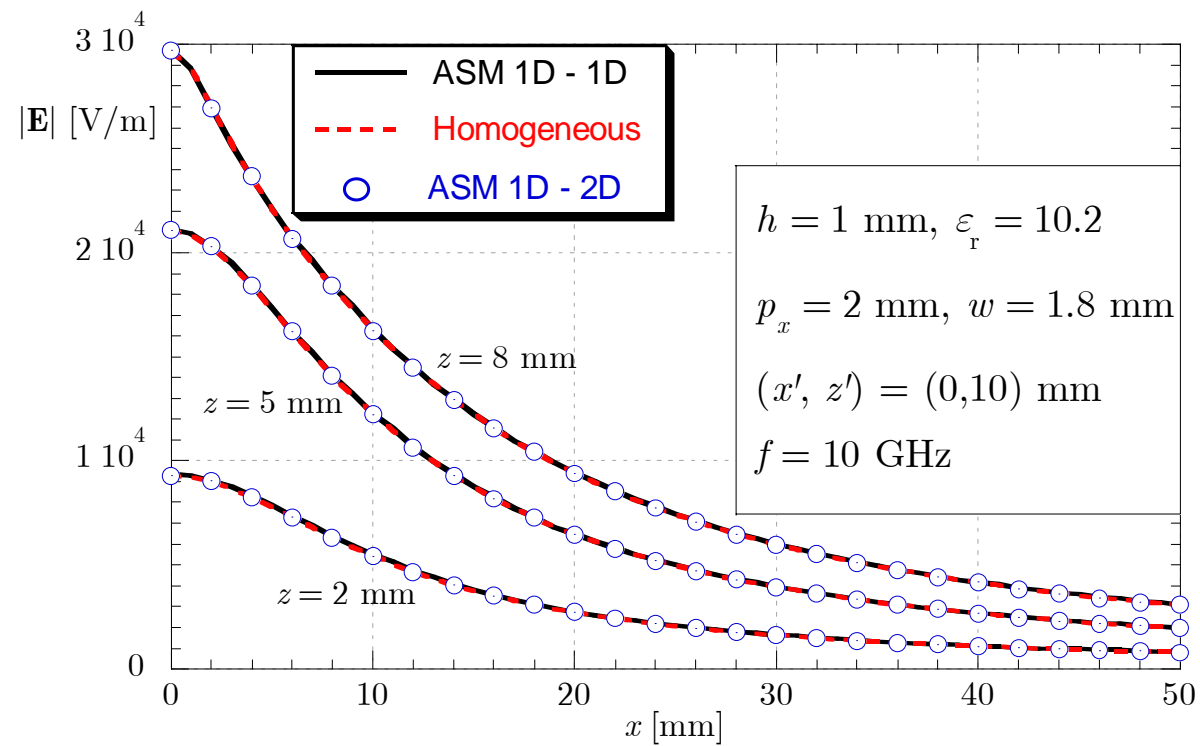
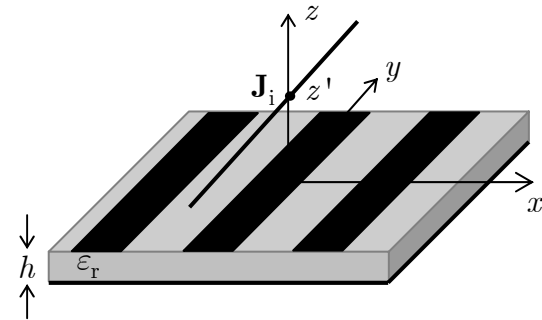
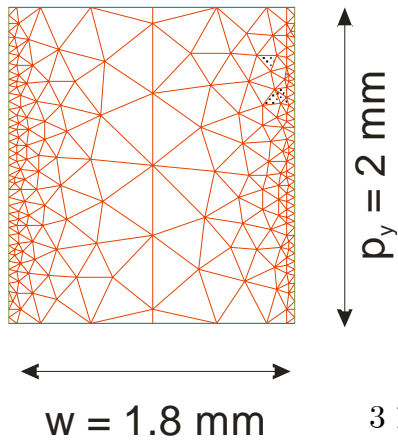
IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, VOL. 53, NO. 1, JANUARY 2005 91

**Fundamental Properties of the Field at the Interface
Between Air and a Periodic Artificial Material
Excited by a Line Source**

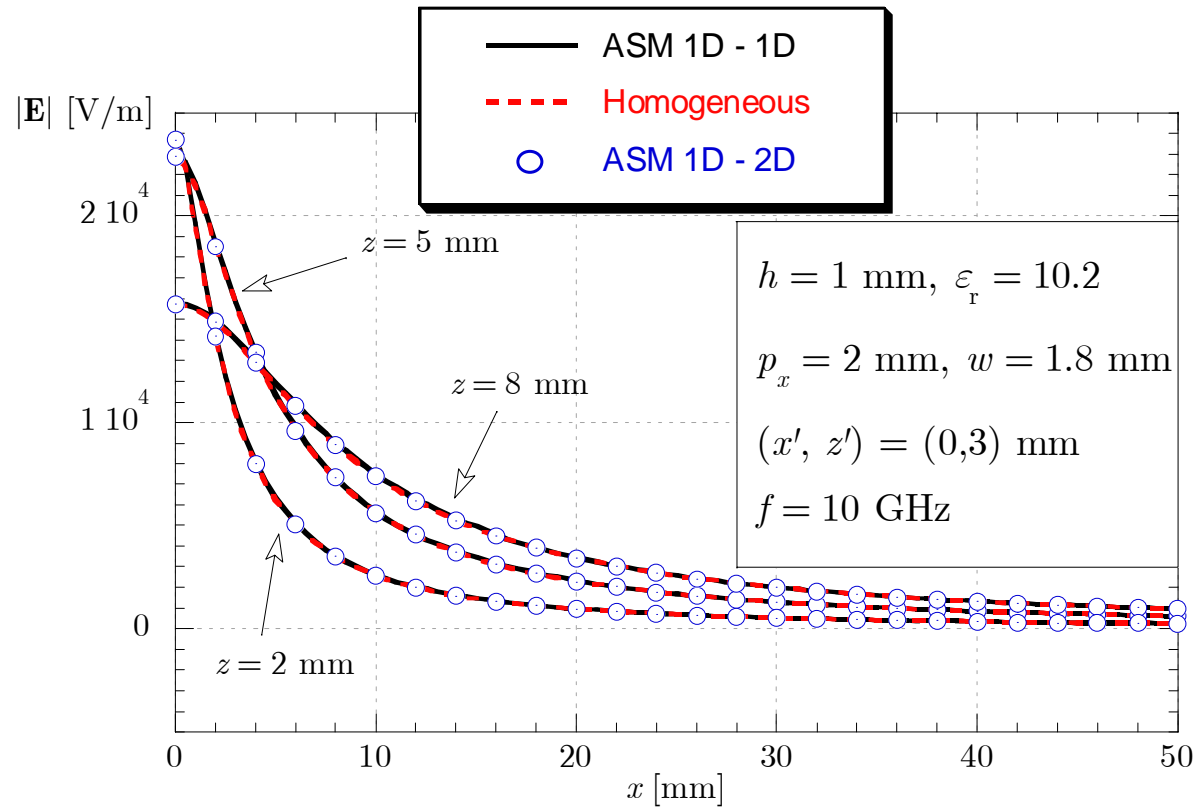
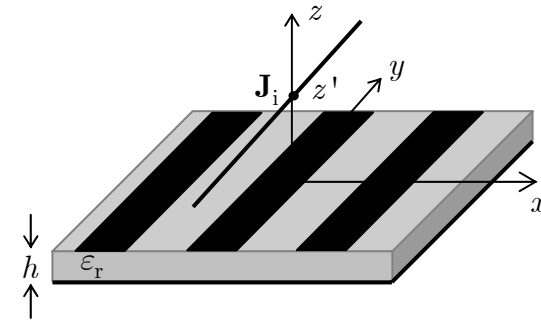
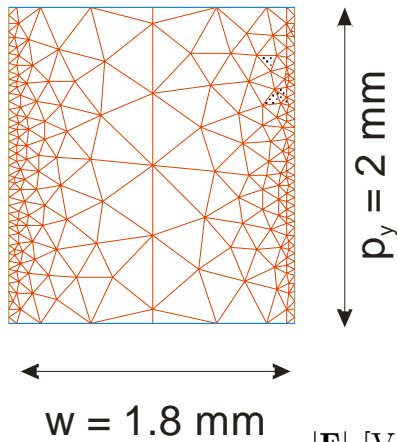
Filippo Capolino, *Senior Member, IEEE*, David R. Jackson, *Fellow, IEEE*, and Donald R. Wilton, *Fellow, IEEE*

The relevant EFIE is discretized in the unit cell expanding the current on the strips through [entire-domain](#) basis functions defined over the strip width and incorporating the appropriate edge singularity.

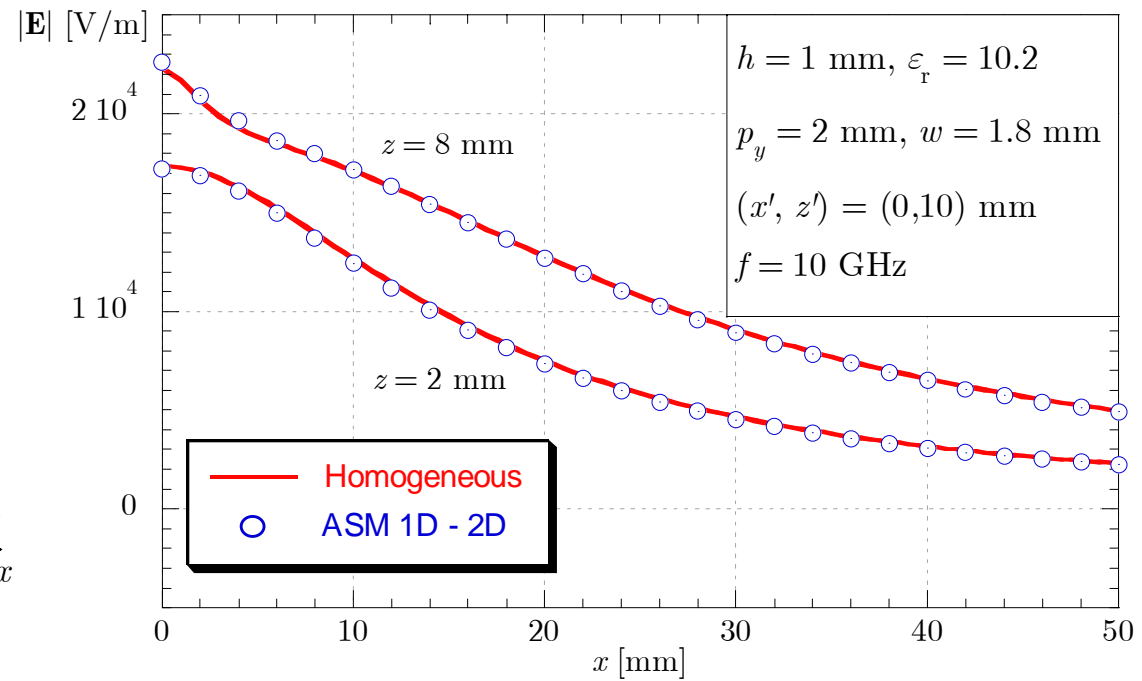
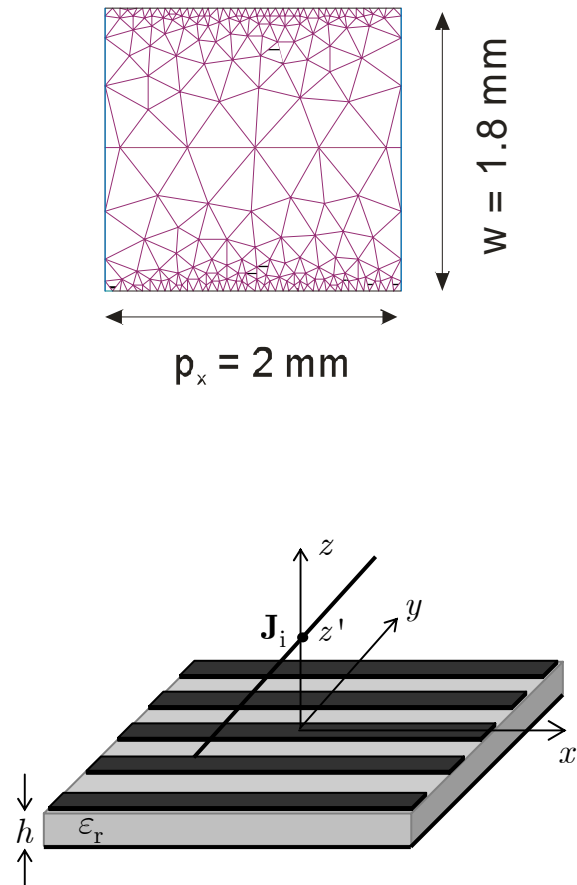
Validation: MSG-GDS parallel (1)



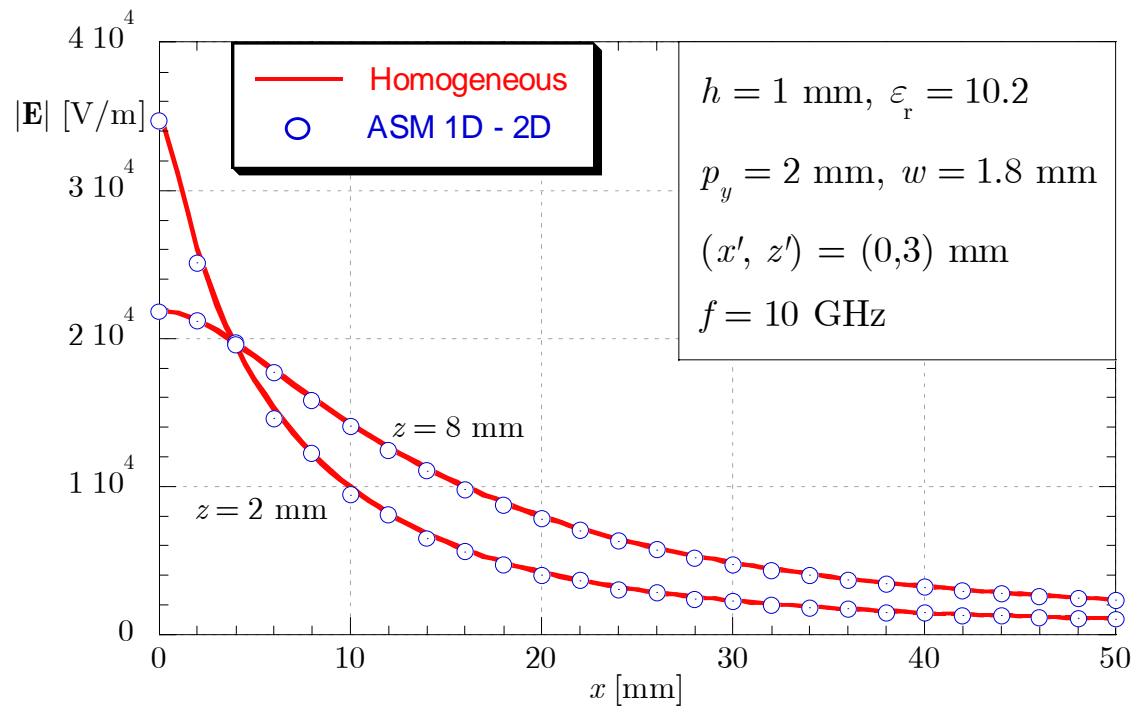
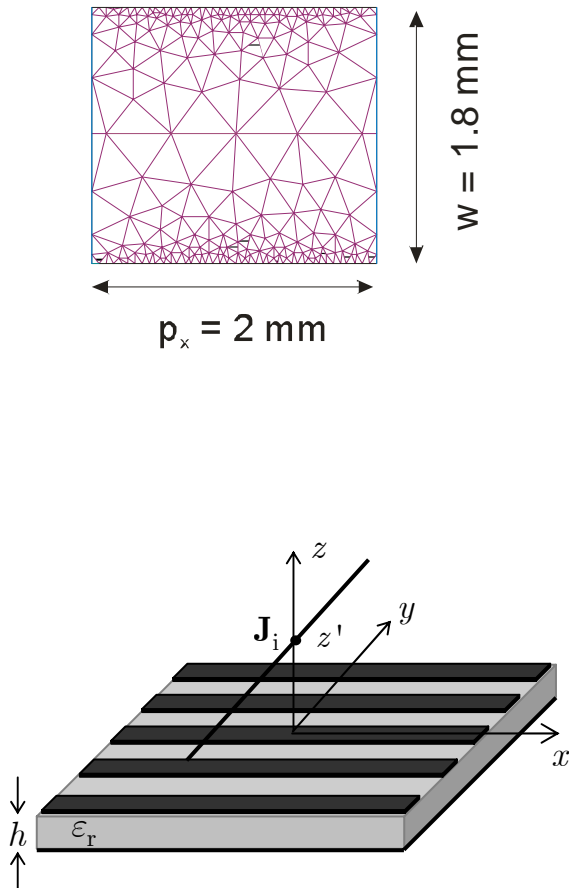
Validation: MSG-GDS parallel (2)



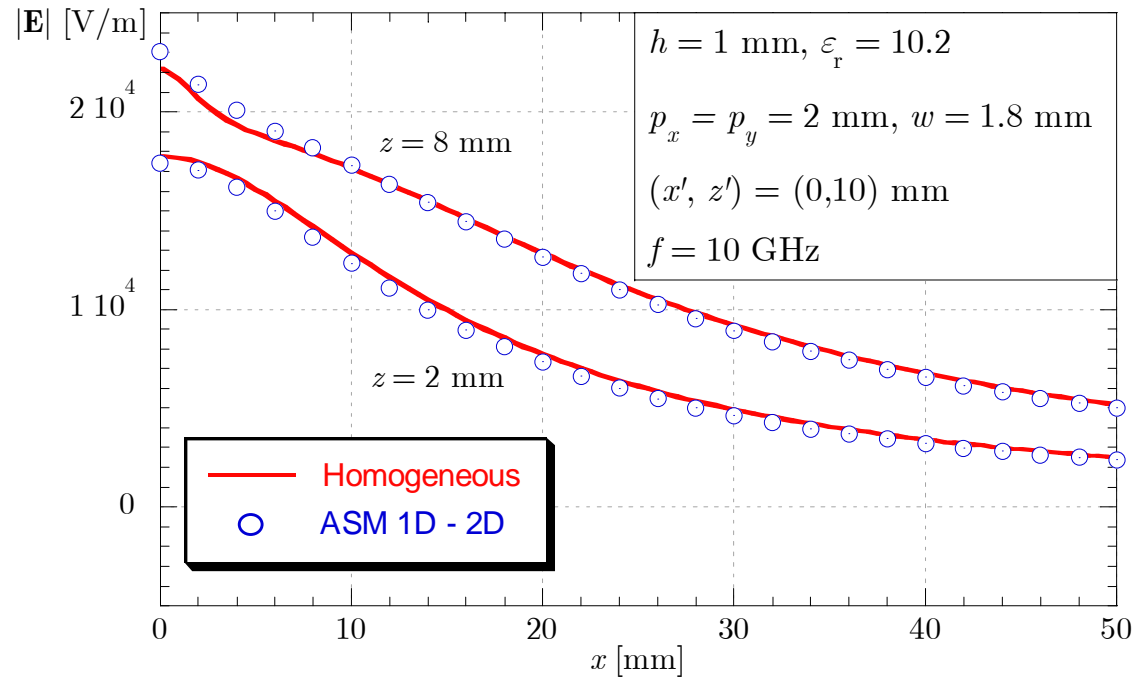
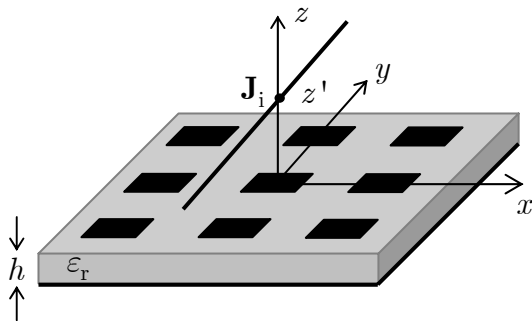
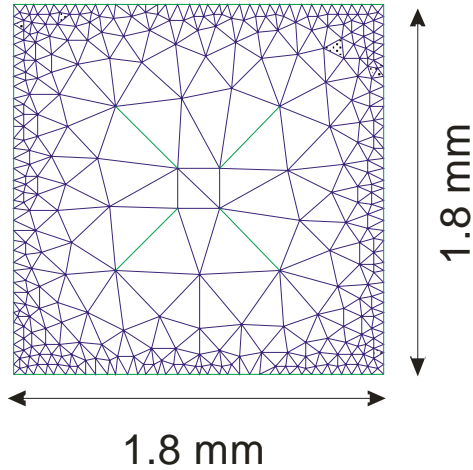
Case Study: MSG-GDS orthogonal (1)



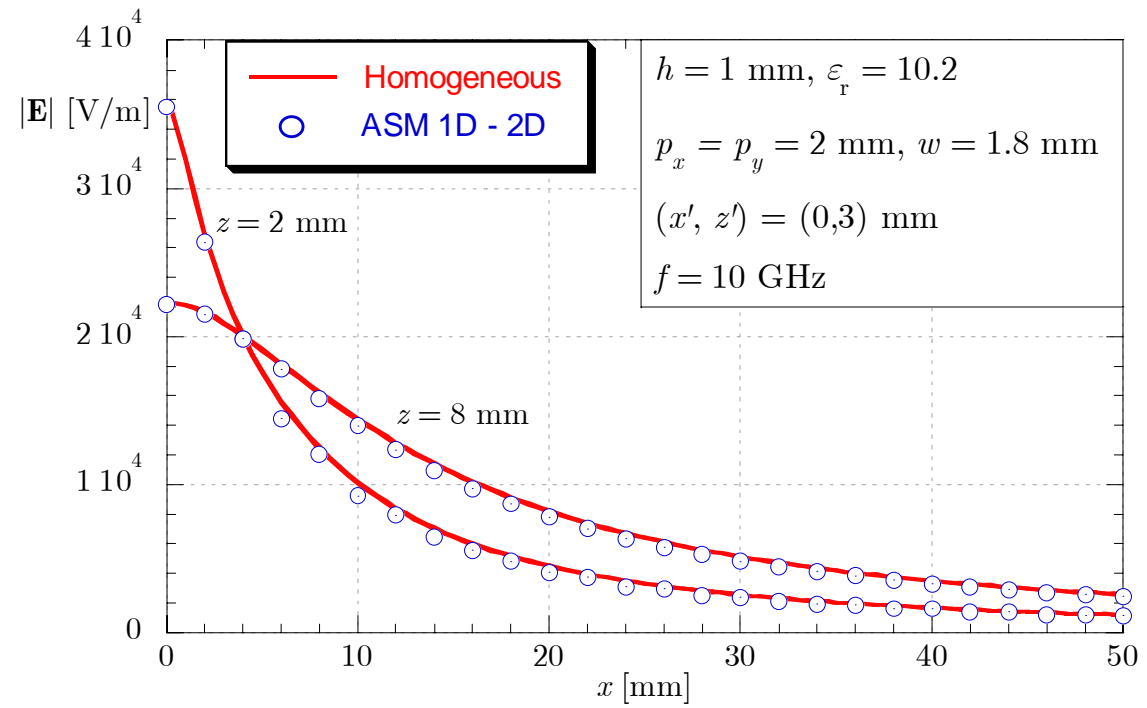
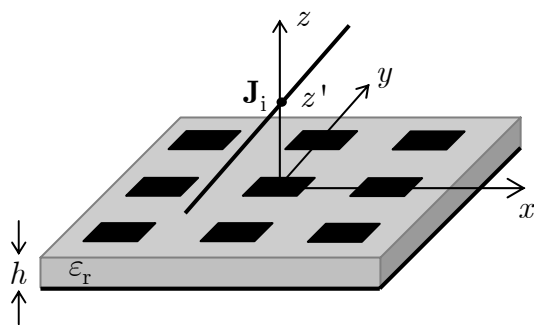
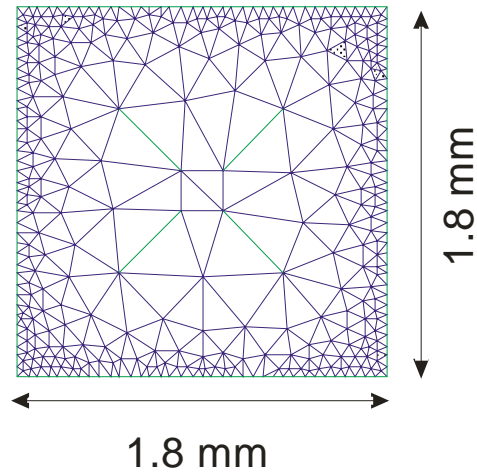
Case Study: MSG-GDS orthogonal (2)



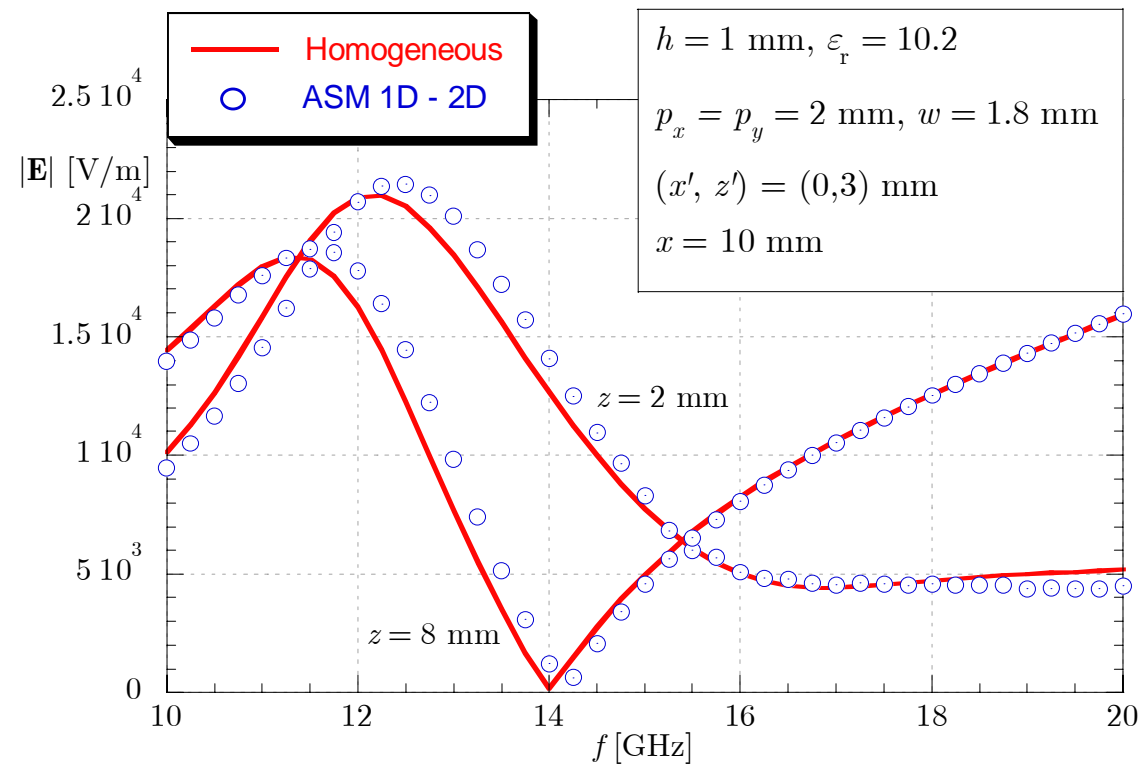
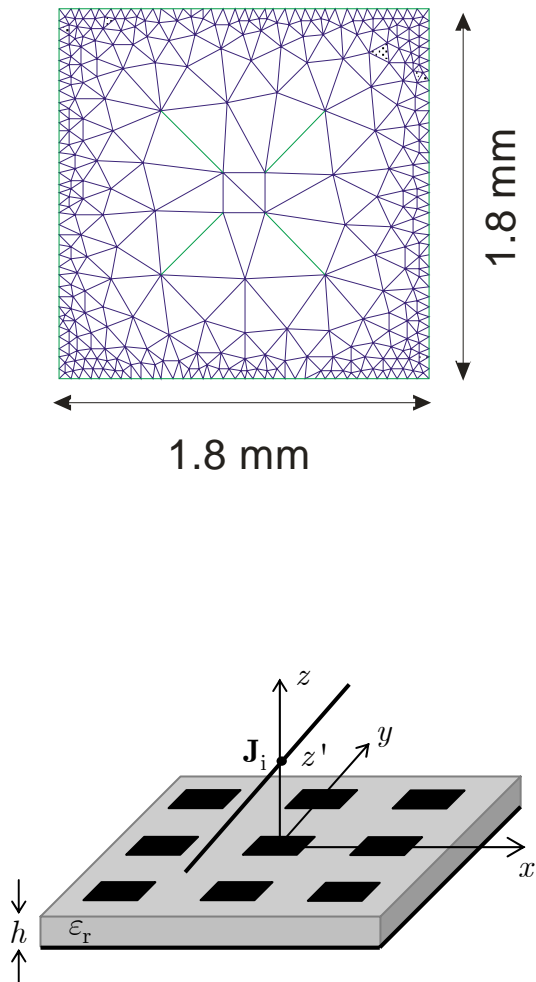
Case Study: Patch Array (1)



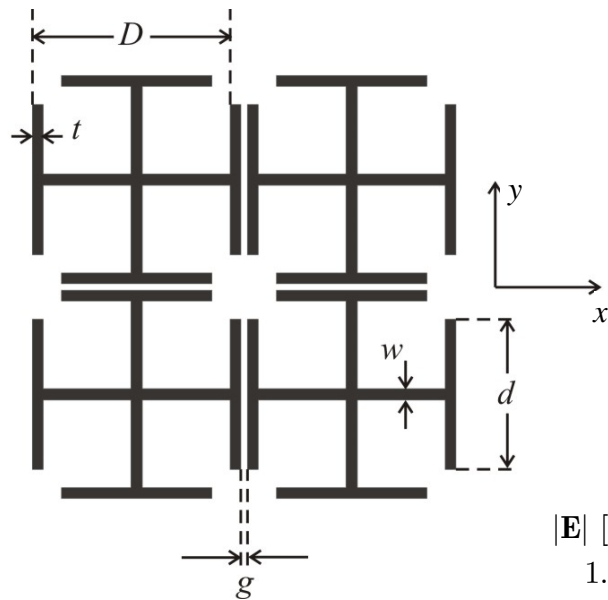
Case Study: Patch Array (2)



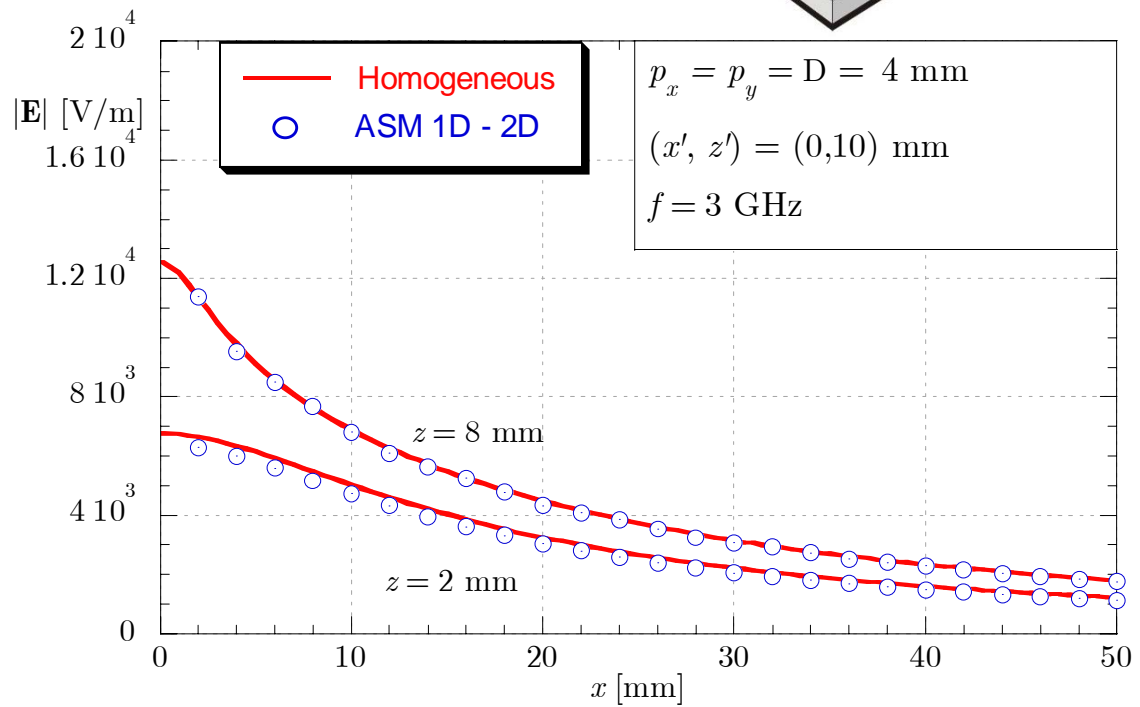
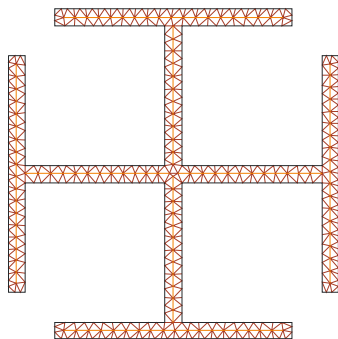
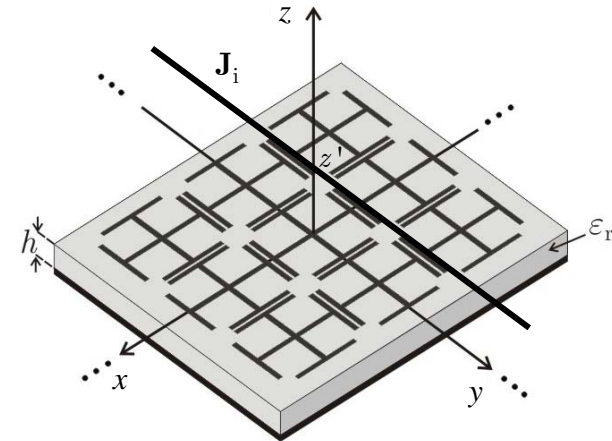
Case Study: Patch Array (3)



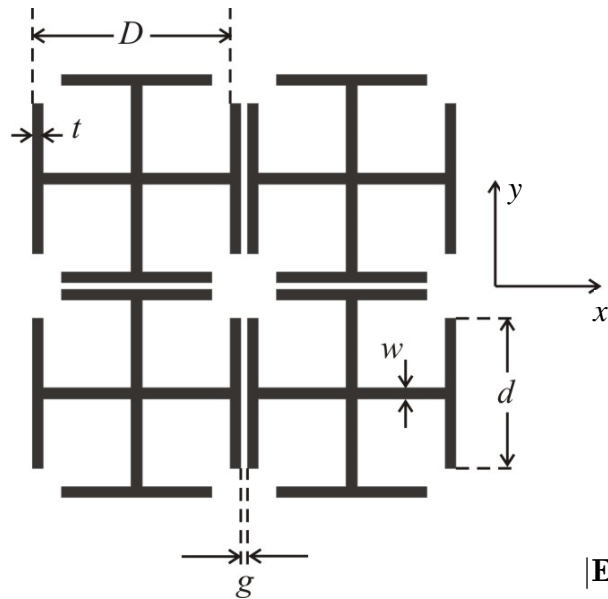
Case Study: Jerusalem Cross (1)



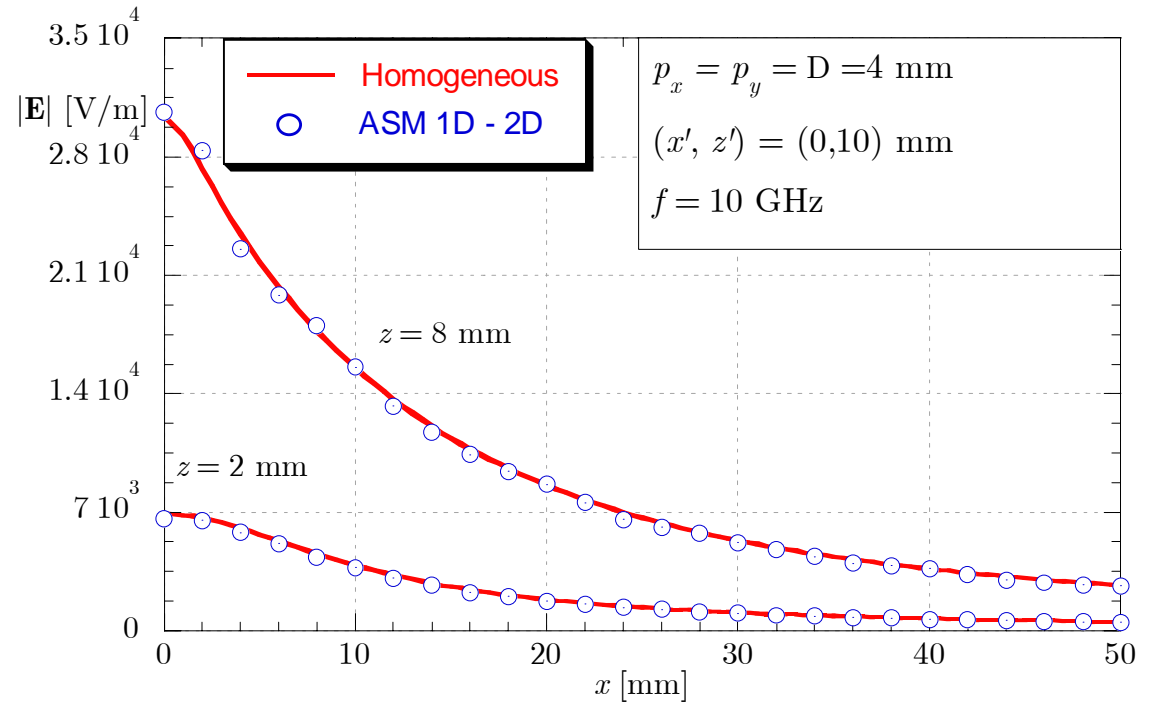
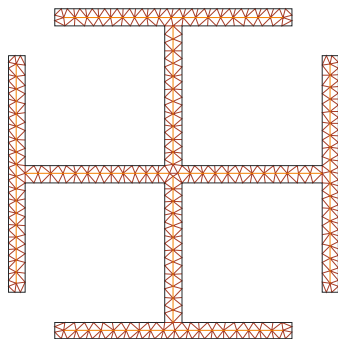
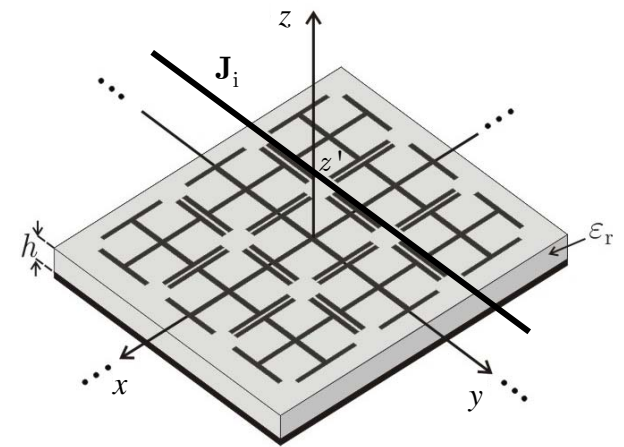
$g = 0.1 \text{ mm}$
 $d = 2.8 \text{ mm}$
 $t = w = 0.2 \text{ mm}$
 $D = 4 \text{ mm}$
 $h = 6 \text{ mm}$
 $\epsilon_r = 2.7$



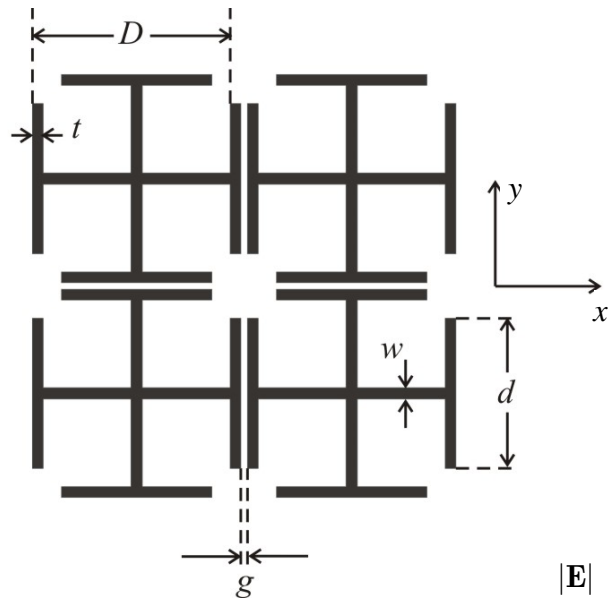
Case Study: Jerusalem Cross (2)



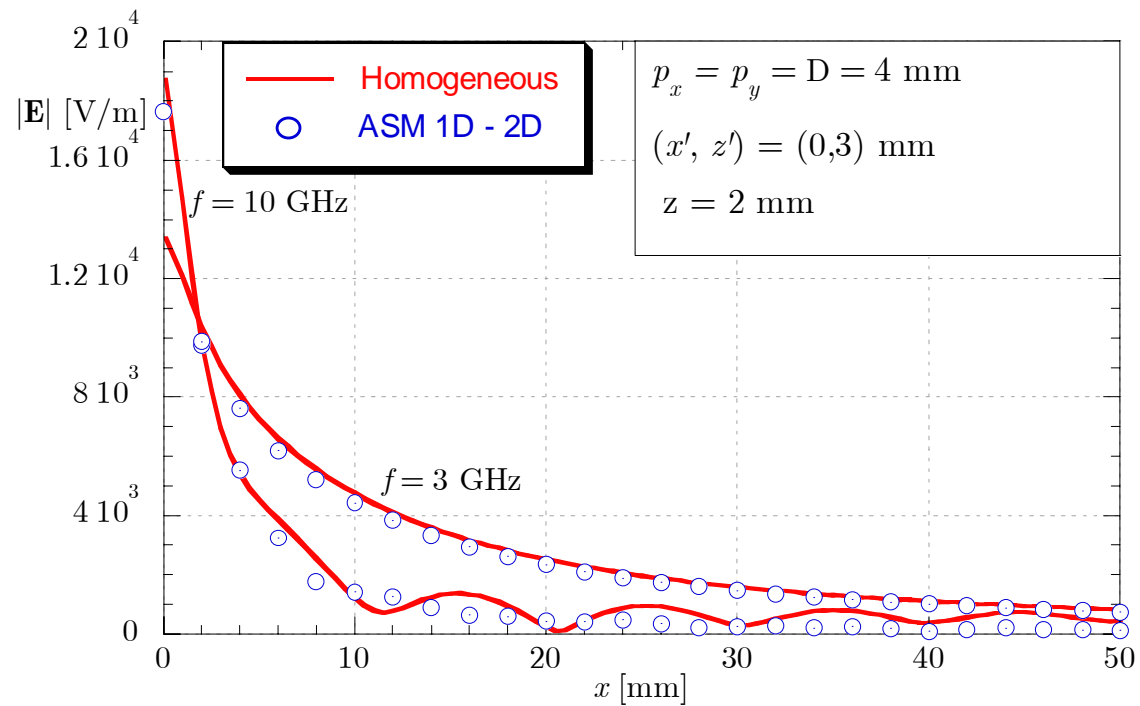
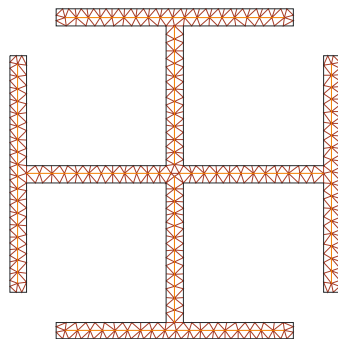
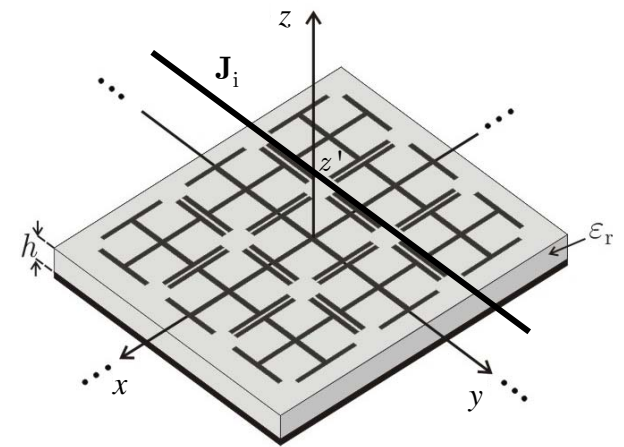
$g = 0.1 \text{ mm}$
 $d = 2.8 \text{ mm}$
 $t = w = 0.2 \text{ mm}$
 $D = 4 \text{ mm}$
 $h = 6 \text{ mm}$
 $\epsilon_r = 2.7$



Case Study: Jerusalem Cross (3)



$g = 0.1 \text{ mm}$
 $d = 2.8 \text{ mm}$
 $t = w = 0.2 \text{ mm}$
 $D = 4 \text{ mm}$
 $h = 6 \text{ mm}$
 $\epsilon_r = 2.7$



Conclusions

Conclusions

- **Array scanning method is developed for the full-wave analysis of HIS structures with 1D and 2D sub-wavelength grids and compared with the homogenized Green's function solution with the electric line source excitation**
- **Near field obtained with the homogenized model and full-wave Method is compared for different source and field points, showing an excellent agreement for several HIS structures**
- **Future work concerns the analysis of near and far fields due to realistic antennas placed in a proximity of HIS structures in order to characterize the conditions of applicability of homogenized Green's function**