Accurate and Rapid Analysis of High-Impedance Surfaces: Plane-Wave and Surface-Wave Analytical Models

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Outline

- Part I

  Analytical Modeling of High Impedance Surfaces

- Part II

  Surface Waves on High Impedance Surfaces

Part I

Outline

- Introduction and Motivation
- Series-Resonant Grid Model
  - ✓ Jerusalem Cross Array
  - ✓ Patch Array
  - ✓ Mushroom Array

Motivation

- HIS structures with electrically small FSS elements
  - ✓ Homogenized FSS grids for far-field and near-field sources
- Metamaterial substrates
  - ✓ Wire media slabs
  - ✓ Slabs with spherical inclusions
- Nanotechnology

Introduction

Analytical modeling of dense FSS grids

- ✓ Homogenization of impedance surface in terms of effective circuit parameters

  Quasi-static approximation of full-wave scattering problem via the averaged impedance boundary condition

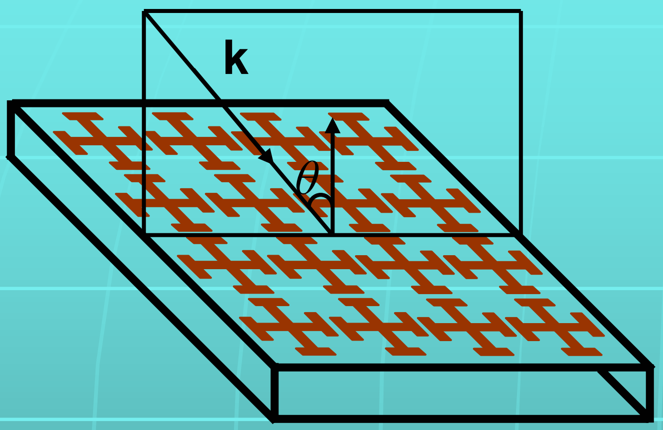
- ✓ Parallel resonance of grid and slab surface impedances

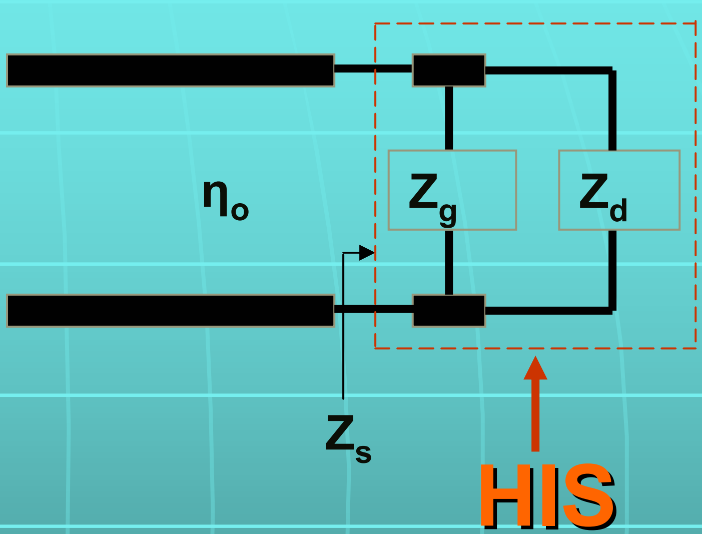
- ✓ Single unit cell of periodic grid and a single Floquet mode

- ✓ Babinet principle

Series-Resonant Grid Model

Transmission-Line Network Analysis





Reflection coefficient

$$\Gamma^{TE}(\omega,\theta)=\frac{Z_s\cos\theta-\eta_0}{Z_s\cos\theta+\eta_0}$$

$$\Gamma^{TM}(\omega,\theta)=\frac{Z_s-\eta_0\cos\theta}{Z_s+\eta_0\cos\theta}$$

$$Z_s=\frac{Z_gZ_d}{Z_g+Z_d}$$

Parallel resonance

$$X_g(\omega)+X_d(\omega)=0$$

S. A. Tretyakov, *Analytical Modeling in Applied Electromagnetics*, Boston, MA: Artech House, 2003

Dielectric Impedance

TE Polarization

$$Z_d^{TE}(\omega,\theta)=\frac{j\eta_0}{\sqrt{\varepsilon_r-\sin^2\theta}}\tan(k_{nd}h)$$

TM Polarization

$$Z_d^{TM}(\omega,\theta)=\frac{j\eta_0}{\sqrt{\varepsilon_r-\sin^2\theta}}\tan(k_{nd}\,h)\left(1-\frac{\sin^2\theta}{\varepsilon_r}\right)$$

Where

$$k_{nd}=\frac{\omega}{c}\sqrt{\varepsilon_r-\sin^2\theta}$$

is the vertical component of the wave vector of the refracted wave

In case of normal incidence

$$Z_d^{TE}(\omega,0)=Z_d^{TM}(\omega,0)=Z_d(\omega,0)$$

- **The dielectric impedance is inductive for all grounded substrates with thickness less than** $\lambda_d/4$

S. A. Tretyakov, *Analytical Modeling in Applied Electromagnetics*, Boston, MA: Artech House, 2003

Grid Impedance

- **The grid impedance is obtained in the quasi-static limit of the full-wave scattering problem via the averaged impedance boundary condition and expressed in terms of effective circuit parameters (effective inductance and effective capacitance)**

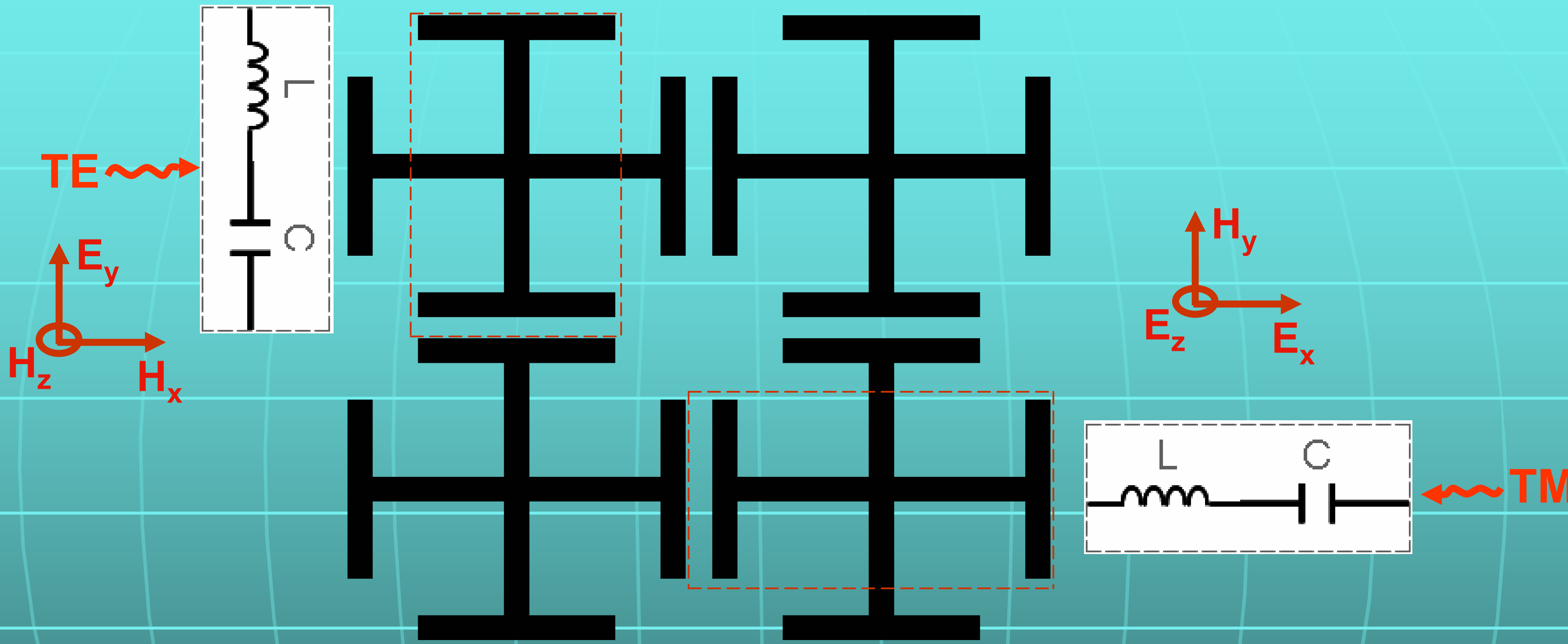
TE Polarization

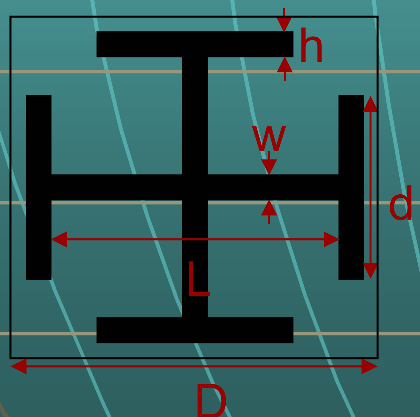
$$Z_g^{TE}(\omega,\theta) = Z_{g,L}^{TE}(\omega,\theta) + Z_{g,C}^{TE}(\omega,\theta)$$

TM Polarization

$$Z_g^{TM}(\omega,\theta) = Z_{g,L}^{TM}(\omega,\theta) + Z_{g,C}^{TM}(\omega,\theta)$$

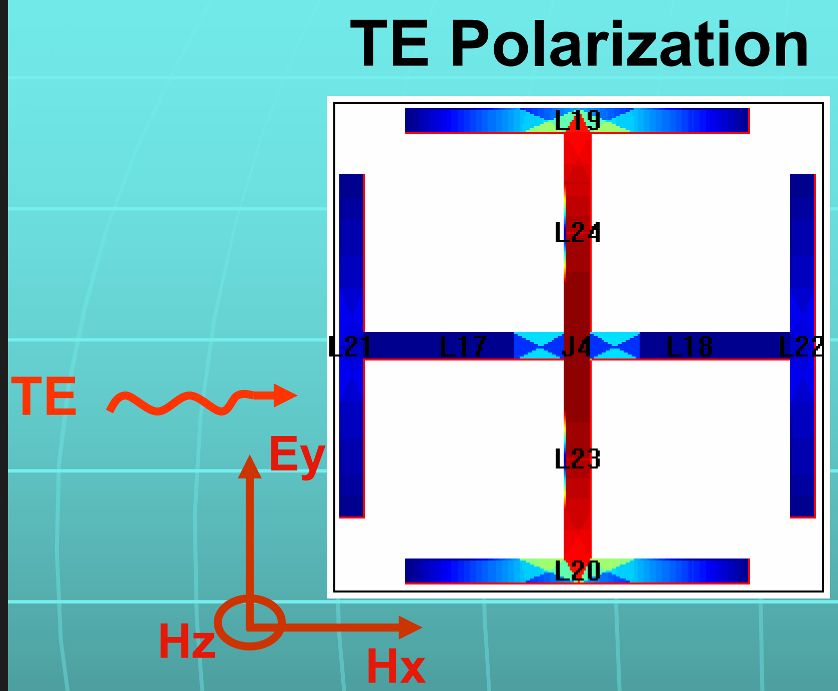
Jerusalem Cross Array

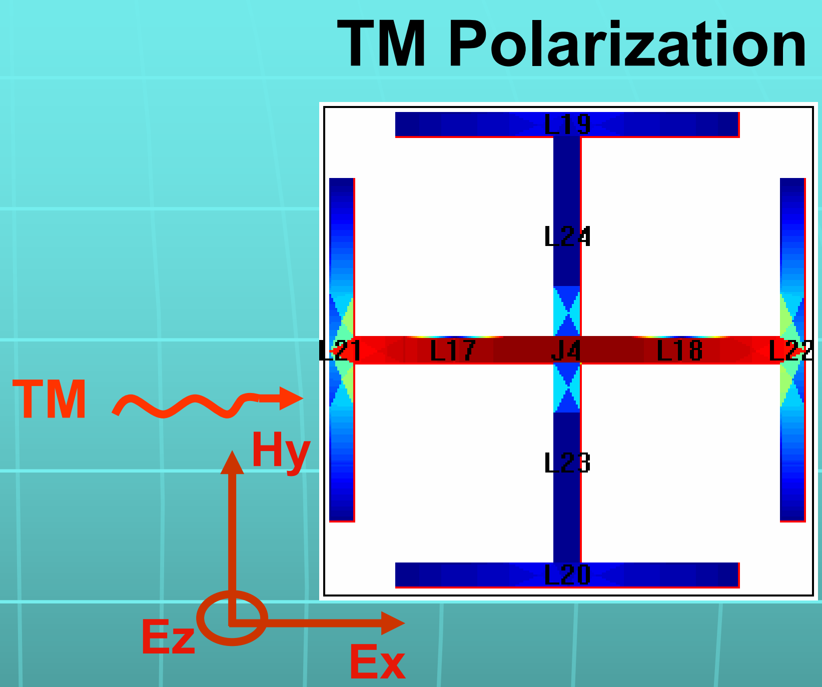




D = 4 mm, L = 3.5 mm, d = 2.8 mm
h = w = 0.2 mm
substrate thickness is 6 mm
dielectric permittivity is 2.7

Current Distribution



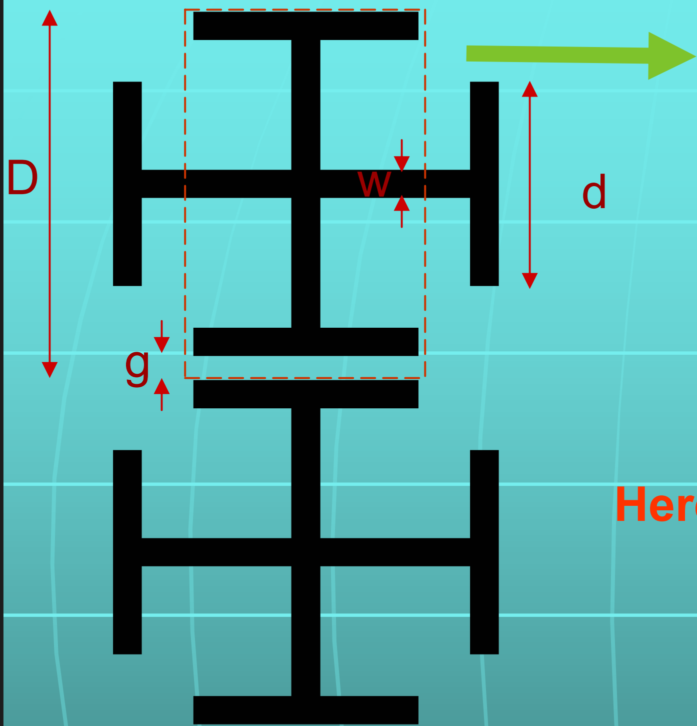


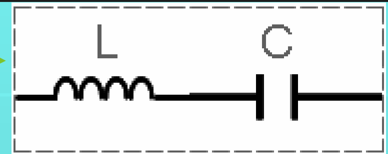
- Current distribution of Jerusalem cross at 4.808 GHz

$$Z_g^{TE}(\omega,\theta) = Z_{g,L}^{TE}(\omega,0) + Z_{g,C}^{TE}(\omega,0)$$

$$Z_g^{TM}(\omega,\theta) = Z_{g,L}^{TM}(\omega,0)\left(1 - 2\frac{\sin^2\theta}{\varepsilon_r + 1}\right) + Z_{g,C}^{TM}(\omega,0)$$

Effective Inductance & Capacitance





$$Z_{g,L}^{TE}(\omega,0) = Z_{g,L}^{TM}(\omega,0) = j\omega L_g$$

$$Z_{g,C}^{TE}(\omega,0) = Z_{g,C}^{TM}(\omega,0) = 1/\,j\omega C_g$$

Here:

$$_g = \frac{\eta_0 \alpha}{2\omega\sqrt{\varepsilon_{eff}}}$$

$$C_g = \frac{1}{\pi}\varepsilon_0\varepsilon_r d\left[\ln\csc\left(\frac{\pi g}{2D}\right) + F\right]$$

Where

$$\alpha = \frac{kD}{\pi}\log\left(\frac{2D}{\pi w}\right)$$

$$F = \frac{Qu^2}{1+Q(1-u)^2} + \left(\frac{du(3u-2)}{4\lambda}\right)^2$$

$$Q = \sqrt{1-\left(\frac{d}{\lambda}\right)^2}$$

$$u = \cos^2\frac{\pi g}{2d}$$

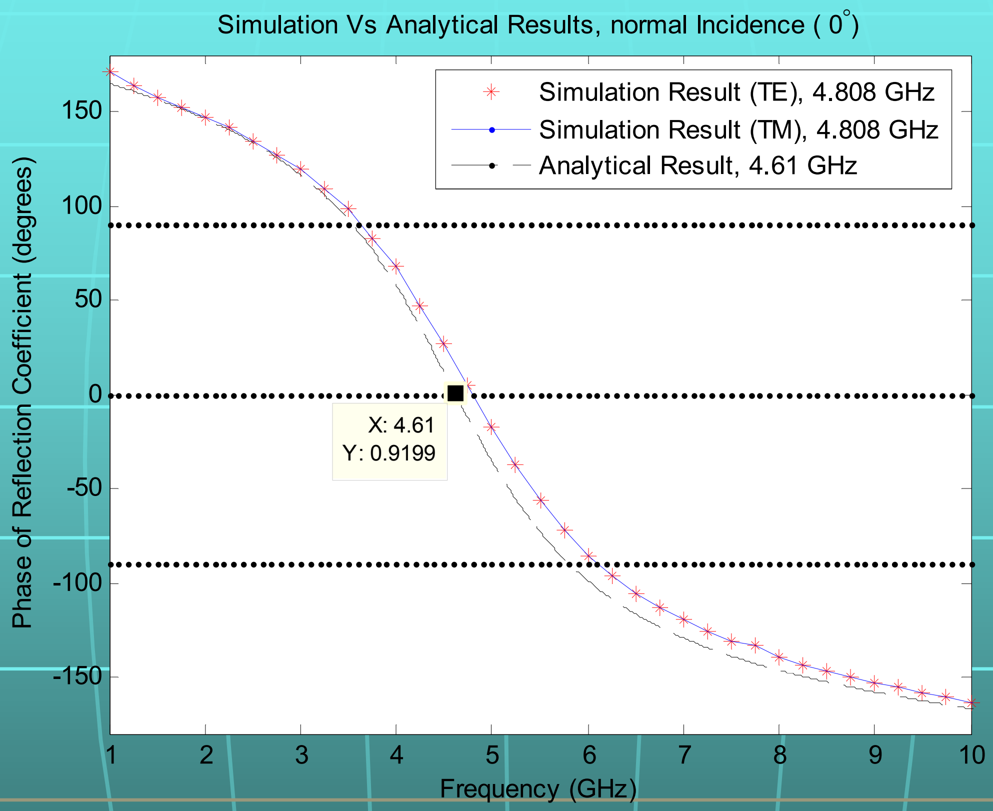
$$\lambda = \frac{2\pi}{k}$$

C. R. Simovski, P. de Maagt, and I. V. Melchakova, "High-impedance surfaces having stable resonance with respect to polarization and incidence angle," *IEEE Trans. Antennas Propagat.*, Vol. 53, no. 3, pp. 908-914, Mar. 2005

S. A. Tretyakov, *Analytical Modeling in Applied Electromagnetics*, Boston, MA: Artech House, 2003

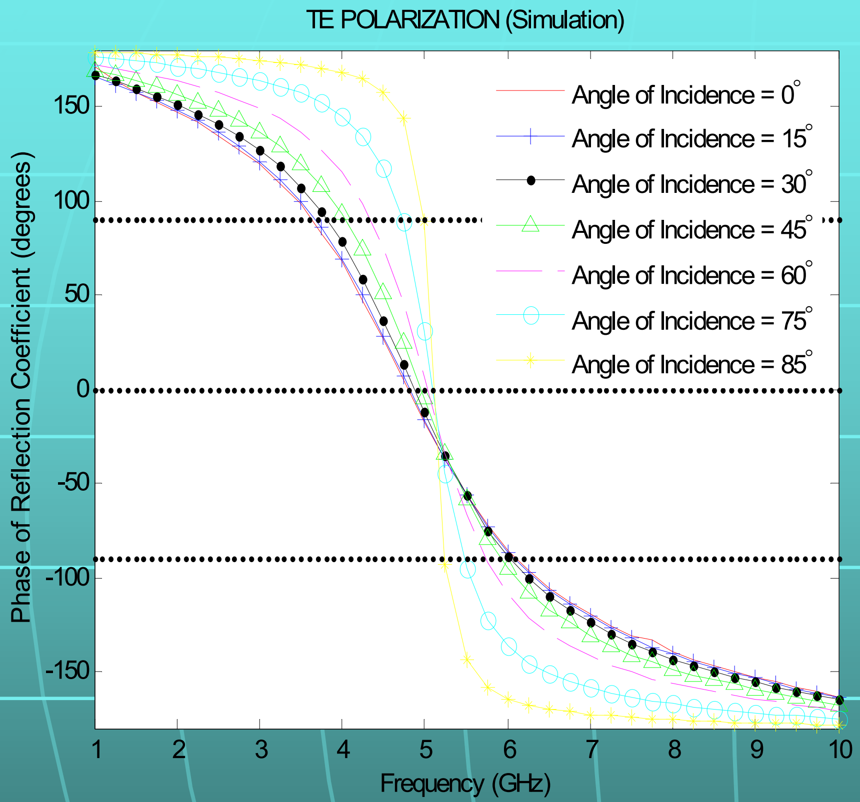
N. Marcuvitz, Waveguide Handbook, Peter Peregrinus Ltd, 1986

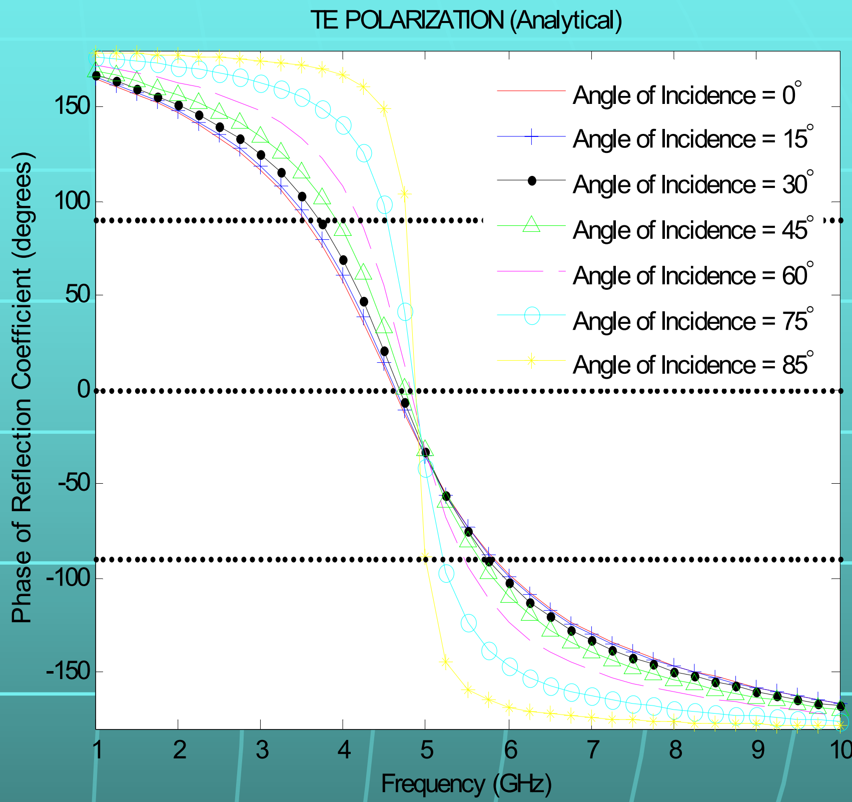
Simulation Vs Analytical Model



Analytical model agrees well with the full wave results by EMPiCASSO

Oblique Incidence (TE Polarization)



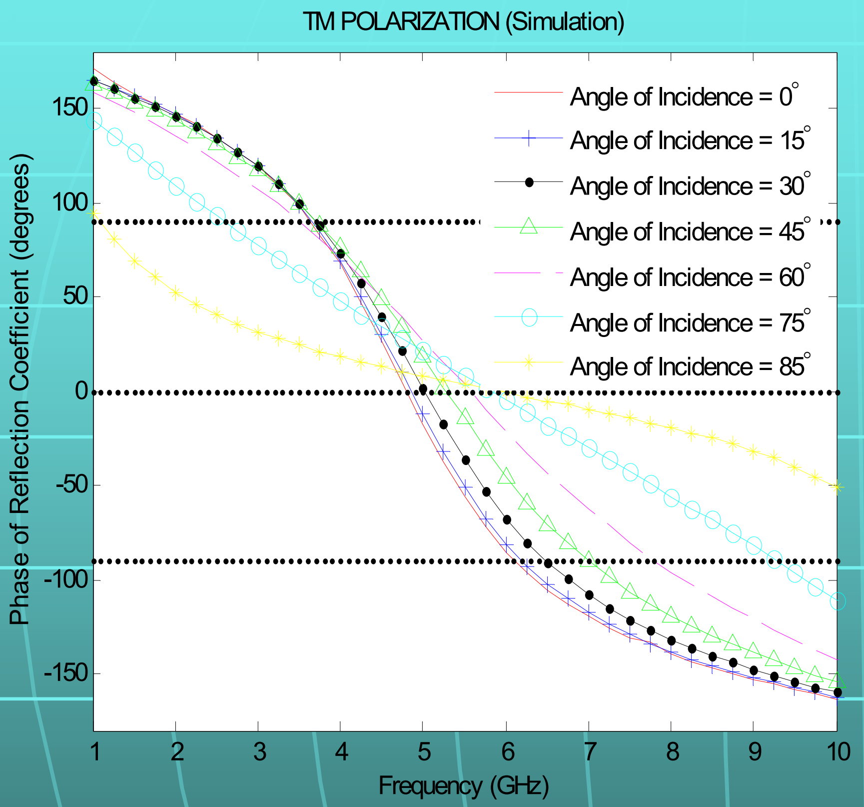


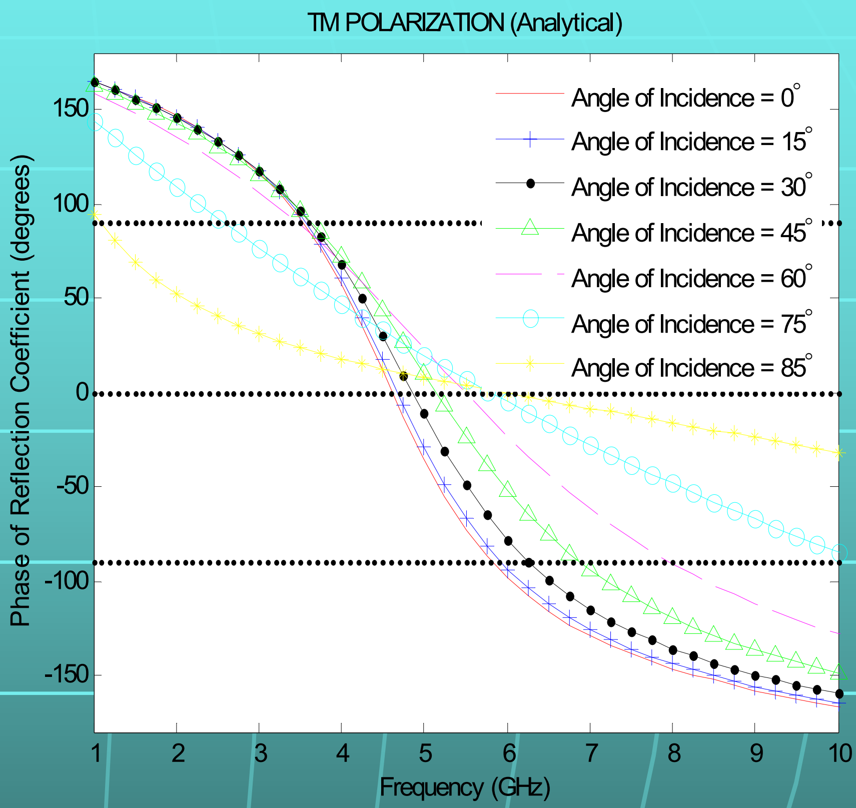
Shift in resonance frequency,

Simulation = (5.122 - 4.808) GHz = 0.314 GHz

Analytical = (4.89 - 4.61) GHz = 0.28 GHz

Oblique Incidence (TM)



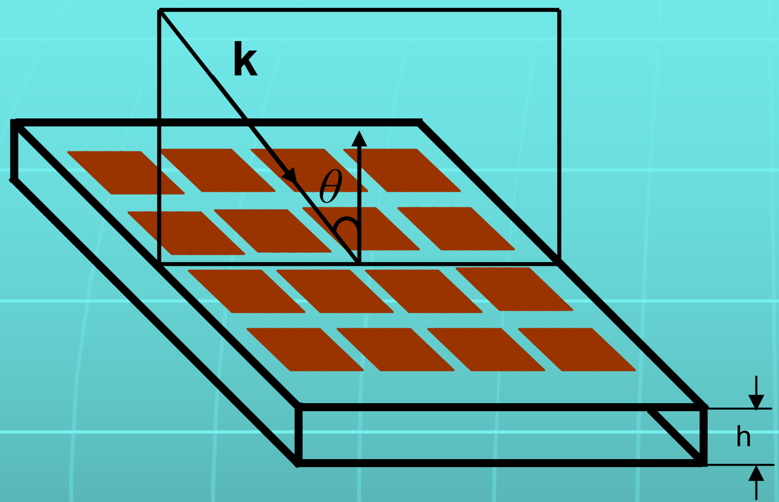


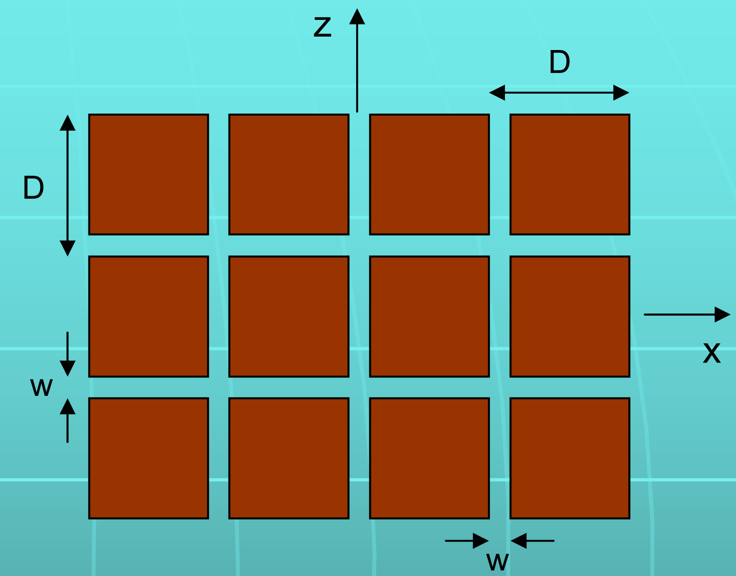
Shift in resonance frequency,

Simulation = (5.885 - 4.808) GHz = 1.077 GHz

Analytical = (5.89 - 4.808) GHz = 1.082 GHz

Patch Array





Grid impedance

- Quasi-static solution of 2D strip grid scattering problem
- Averaged impedance boundary condition
- Approximate Babinet principle

$$Z_g^{TE} = -j\frac{\eta_{eff}}{2\alpha}\frac{1}{\left(1-\frac{1}{2}\frac{k_0^2}{k_{eff}^2}\sin^2\theta\right)}$$

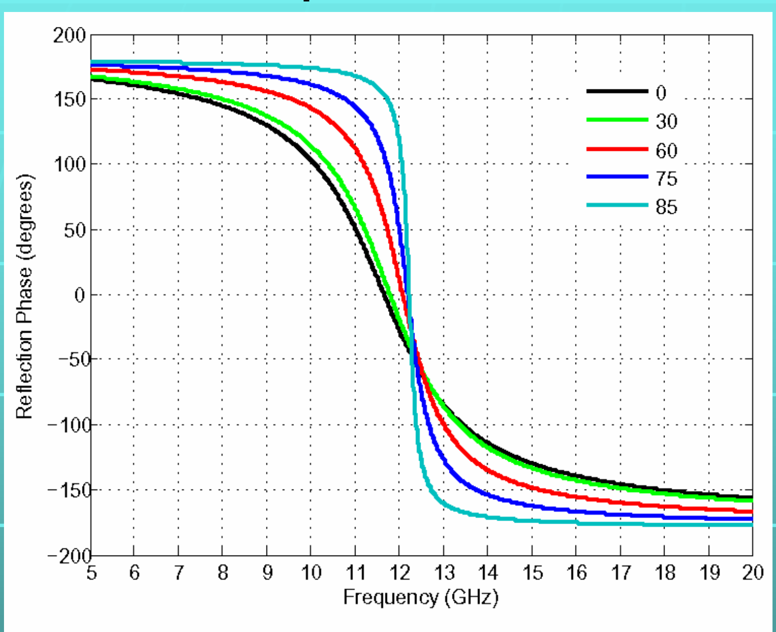
$$Z_{gc}^{TM} = -j\frac{\eta_{eff}}{2\alpha}$$

$$\alpha = \frac{k_{eff}D}{\pi}\ln\left(\csc\left(\frac{\pi w}{2D}\right)\right)$$

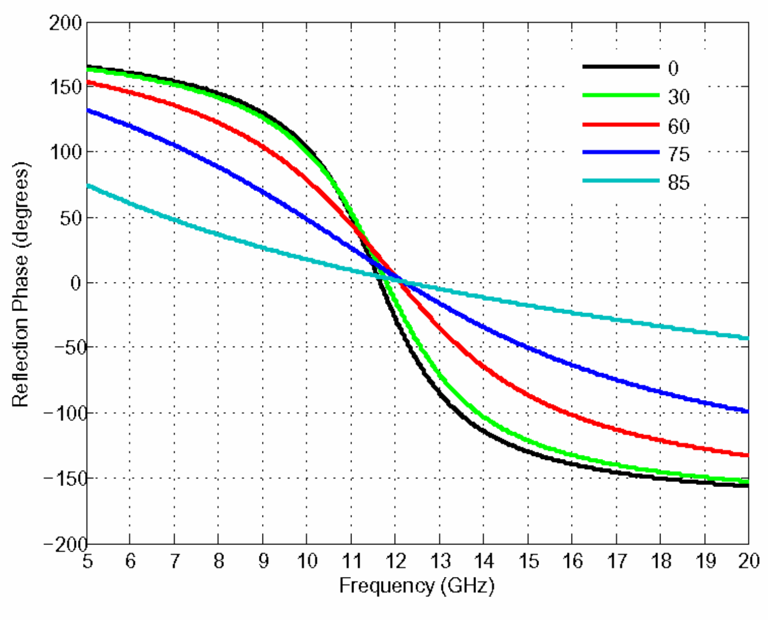
O. Luukkonen, C. R. Simovski, G. Granet, G. Goussetis, D. Lioubtchenko, A. V. Raisanen, and S. A. Tretyakov, "Simple and accurate analytical model of planar grids and high-impedance surfaces comprising metal strips or patches, http://arxiv.org/abs/0705.3548.

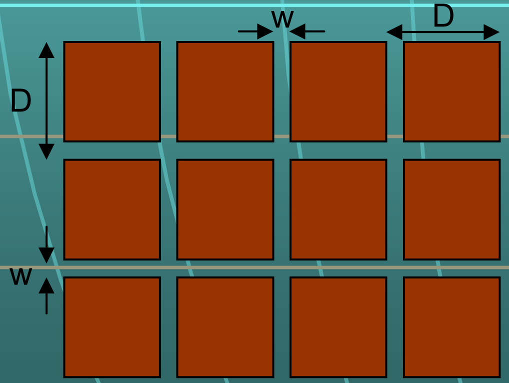
Oblique Incidence

TE polarization



TM polarization

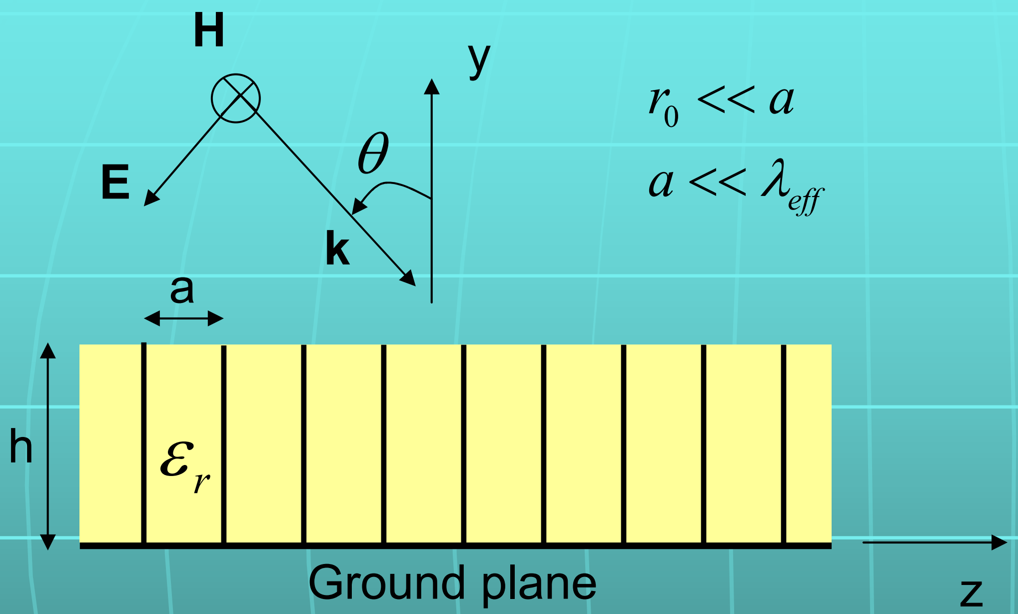




D = 2 mm, w = 0.2 mm
substrate thickness is 1 mm
dielectric permittivity is 10.2

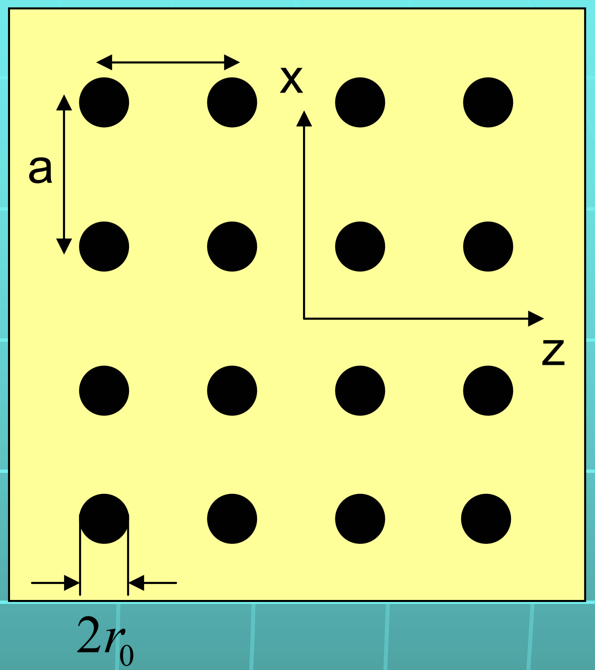
O. Luukkonen, C. R. Simovski, G. Granet, G. Goussetis, D. Lioubtchenko, A. V. Raisanen, and S. A. Tretyakov, “Simple and accurate analytical model of planar grids and high-impedance surfaces comprising metal strips or patches, http://arxiv.org/abs/0705.3548.

Wire Media Slab



$r_0 << a$

$a << \lambda_{eff}$



Anisotropic material characterized by effective permittivity

Quasi-static approximation

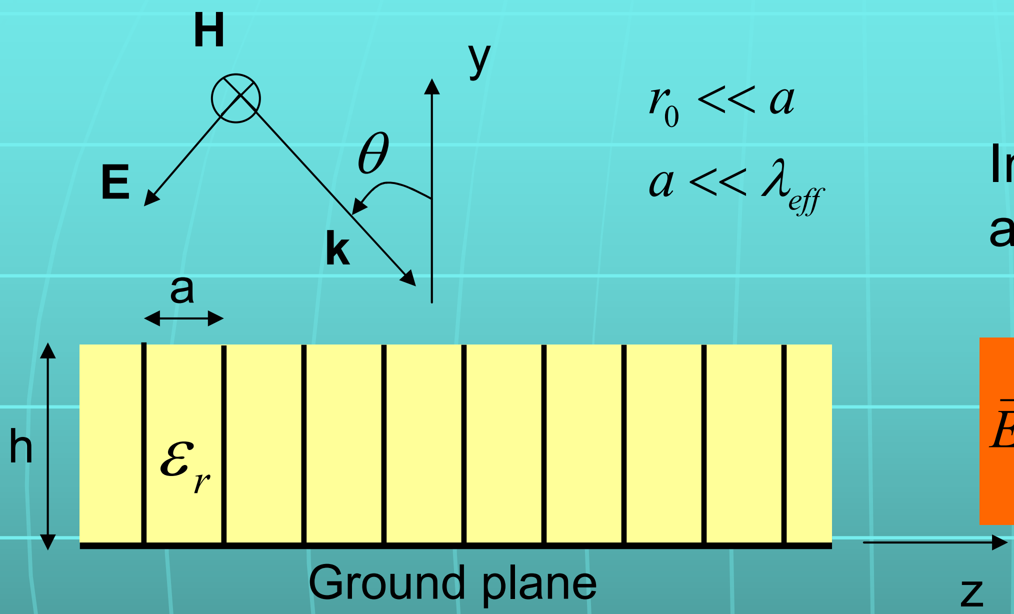
$$\overleftrightarrow{\varepsilon}_{eff} = \varepsilon_0 \varepsilon_r \left(\hat{x}\hat{x} + \varepsilon_{yy} \hat{y}\hat{y} + \hat{z}\hat{z} \right)$$

$$\varepsilon_{yy} = 1 - \frac{k_p^2}{k_0^2 \varepsilon_r}$$

k_p is the plasma wavenumber

$$k_p^2 = \frac{2\pi / a^2}{\ln\left(\frac{a^2}{4r_0(a - r_0)} \right)}$$

Surface Impedance of Wire Media Slab



$r_0 << a$

$a << \lambda_{eff}$

Impedance boundary condition at y=h:

$$\vec{E}_t = j\omega\mu_0 \frac{\tan\left(k_{yd}h\right)}{k_{yd}}\left(1-\frac{k_z^2}{k_0^2\varepsilon_{yy}}\right)\hat{n}\times\vec{H}_t$$

Surface impedance

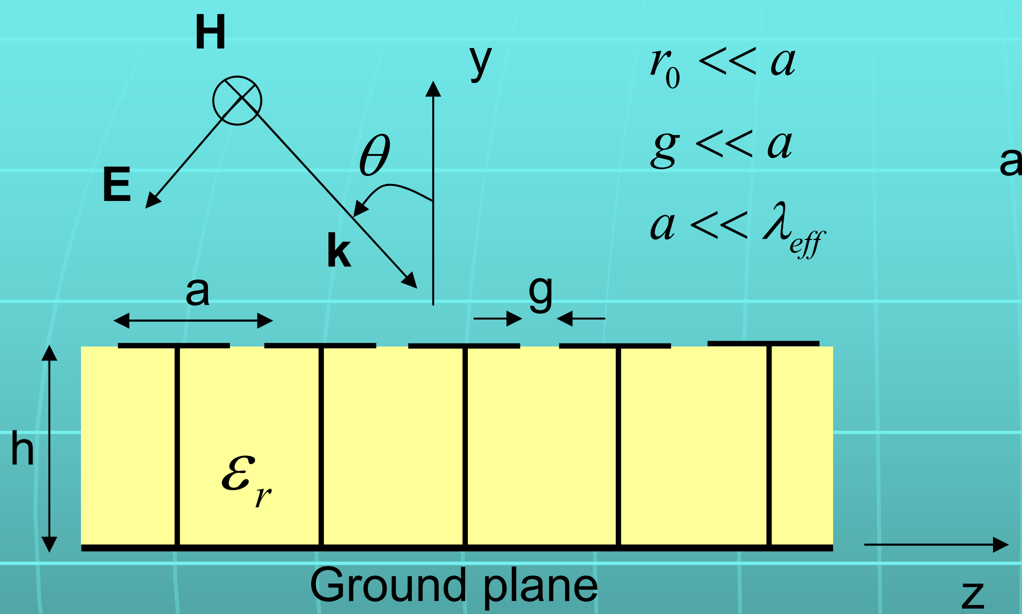
$$Z_d^{TM} = j\omega\mu_0 \frac{\tan\left(k_{yd}h\right)}{k_{yd}} \frac{k_0^2\varepsilon_r - k_p^2 - k_z^2}{k_0^2\varepsilon_r - k_p^2}$$

$$k_{yd} = \sqrt{k_0^2\varepsilon_r - \frac{\varepsilon_r}{\varepsilon_{yy}}k_z^2}$$

$$\varepsilon_{yy} = 1 - \frac{k_p^2}{k_0^2\varepsilon_r}$$

S. A. Tretyakov, *Analytical Modeling in Applied Electromagnetics*, Boston, MA: Artech House, 2003

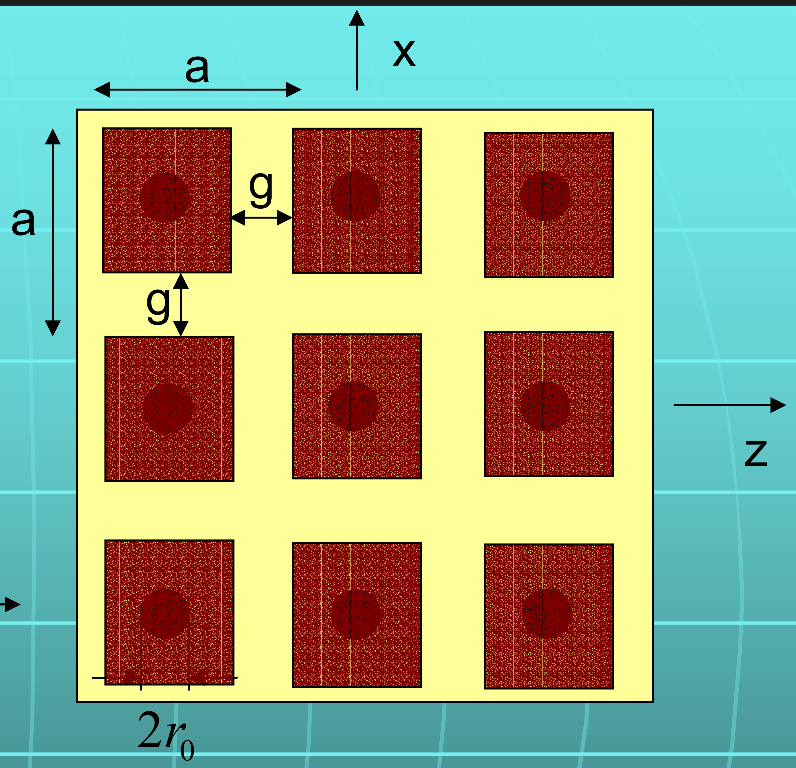
Mushroom Array

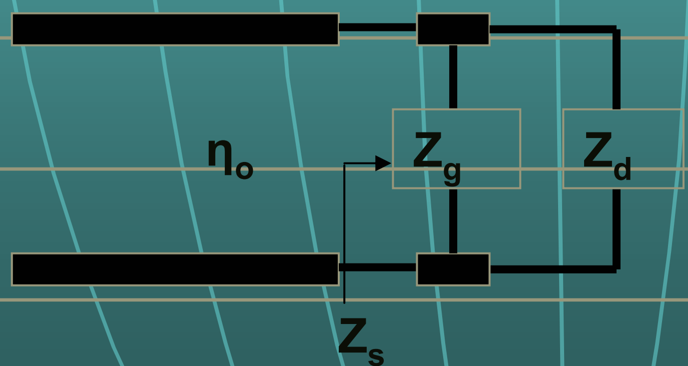


$$r_0 << a$$

$$g << a$$

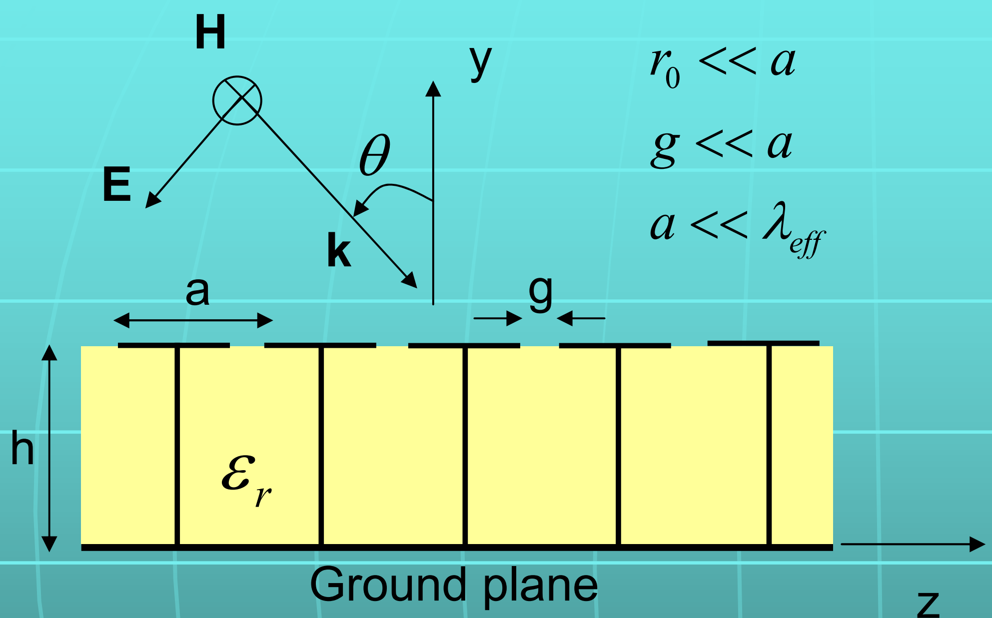
$$a << \lambda_{eff}$$





$$Z_s = \frac{Z_g Z_d}{Z_g + Z_d}$$

Mushroom Array

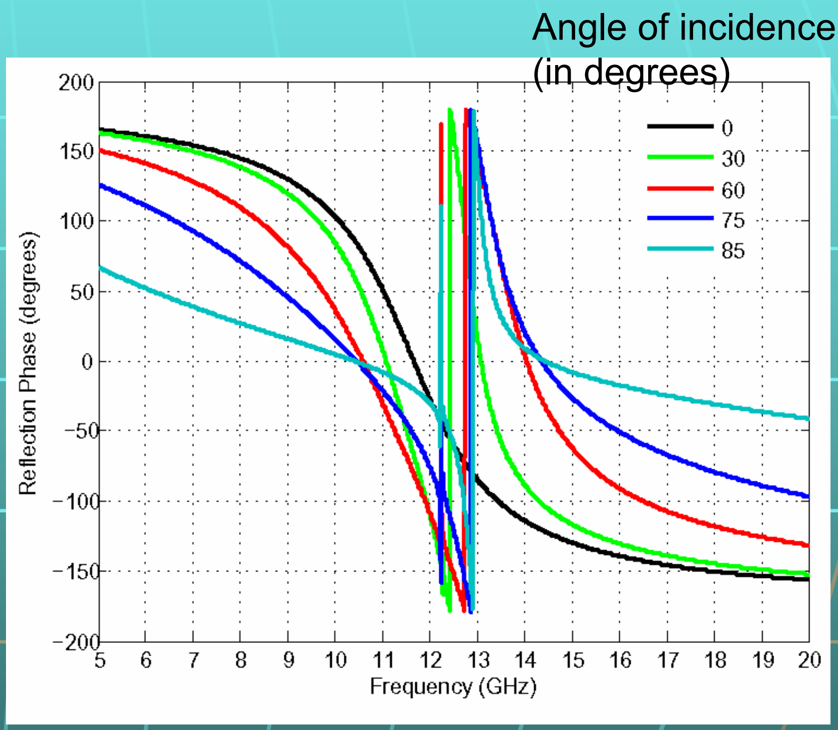


$r_0 << a$

$g << a$

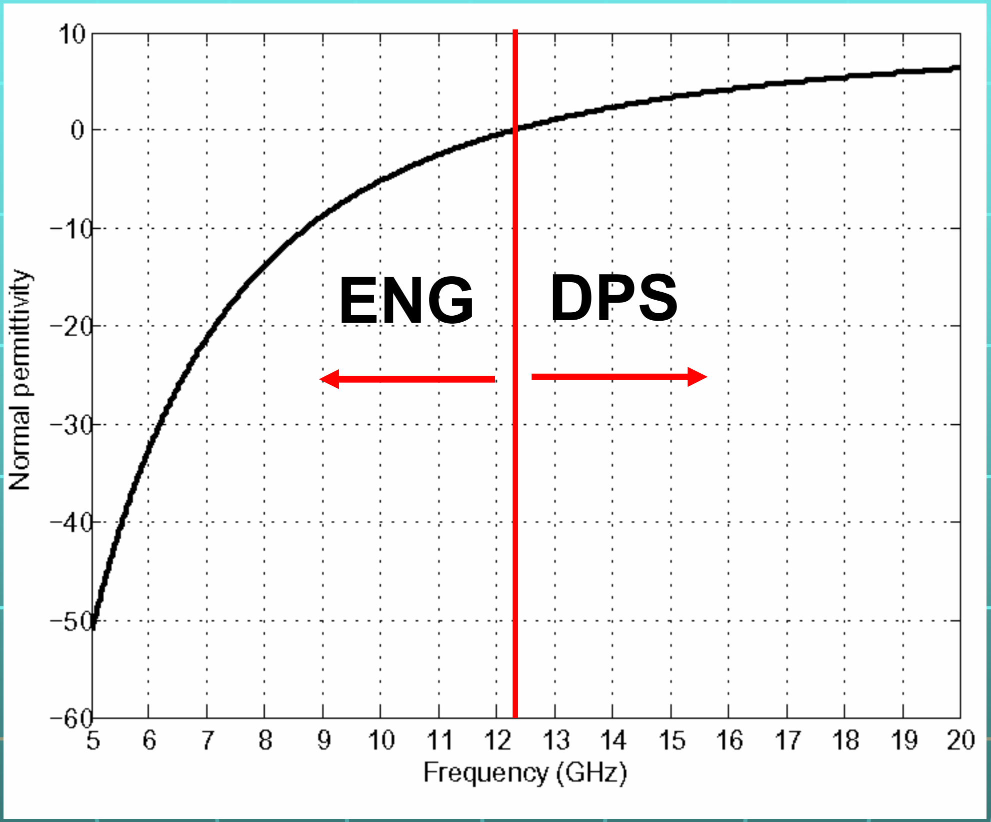
$a << \lambda_{eff}$

TM polarization



Period of vias is 2 mm
Period of patches is 2 mm
Gap is 0.2 mm
Radius of vias is 0.05 mm
Substrate thickness is 1 mm
Dielectric permittivity is 10.2

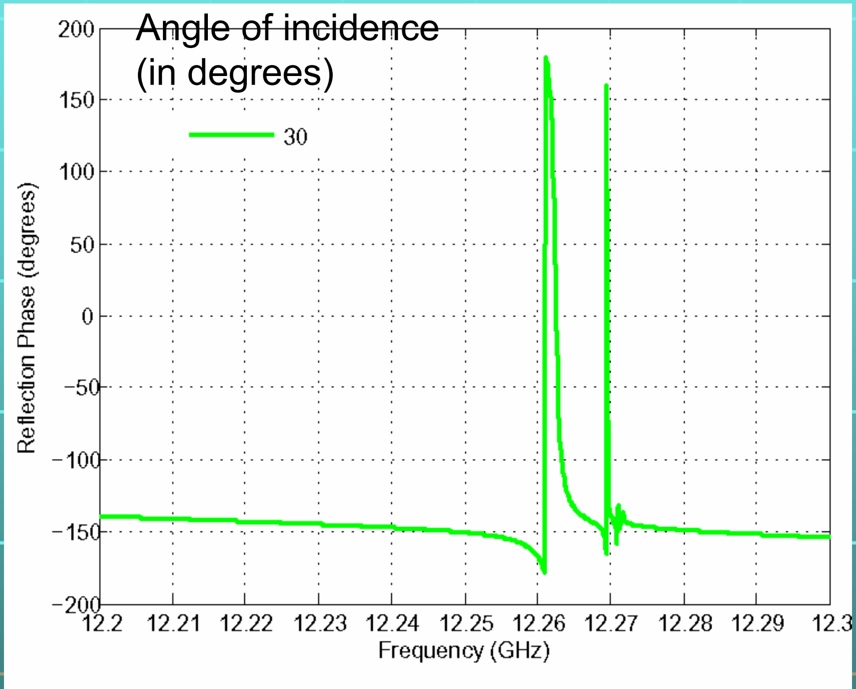
Effective Permittivity Along the Vias

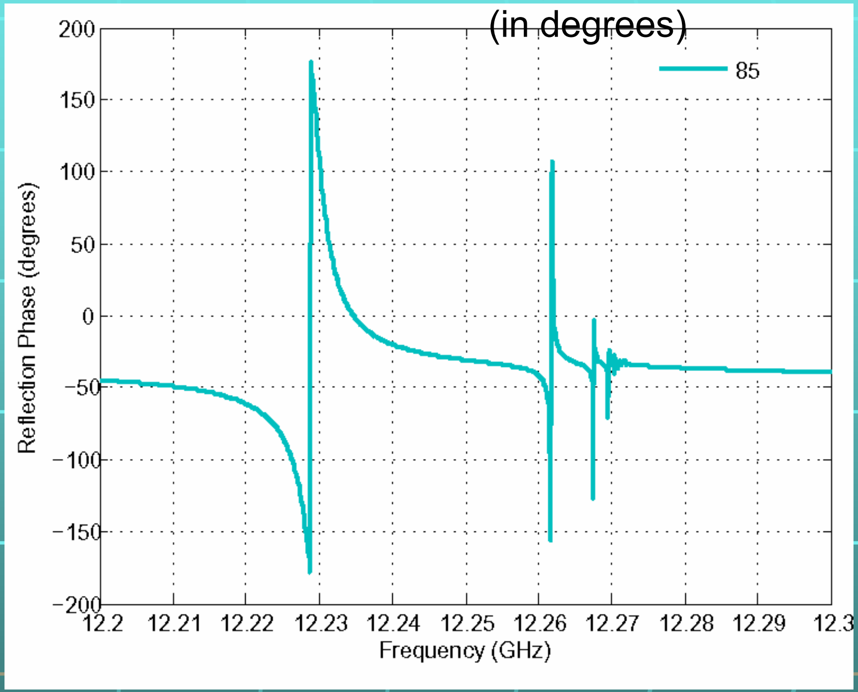


Wire media slabs and mushroom structures support backward waves in the ENG frequency band

Plane-Wave Interaction

TM polarization





Multiple resonances occur in a narrow frequency band close to the transition of effective media from ENG to DPS. These resonances are due to coupling of plane-wave to higher-order modes excited close to the transition point.

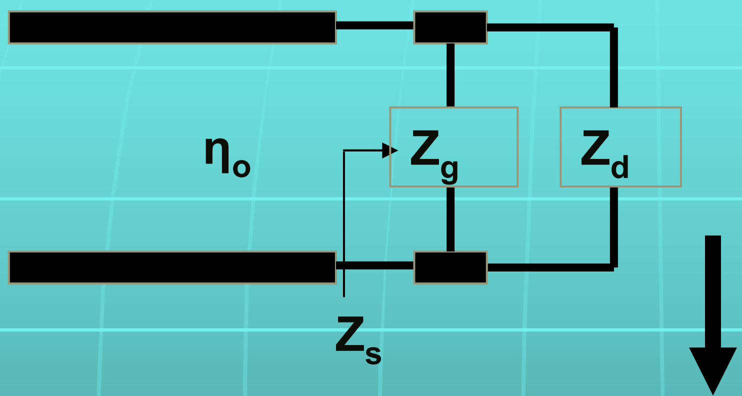
Part II

Outline

- Model 1 – Impedance Surface
- Model 2 – Grounded Dielectric Slab with Grid Impedance
  - ✓ Dispersion equations for surface waves
  - ✓ Dispersion behavior of proper and improper TE and TM surface waves
- Conclusion

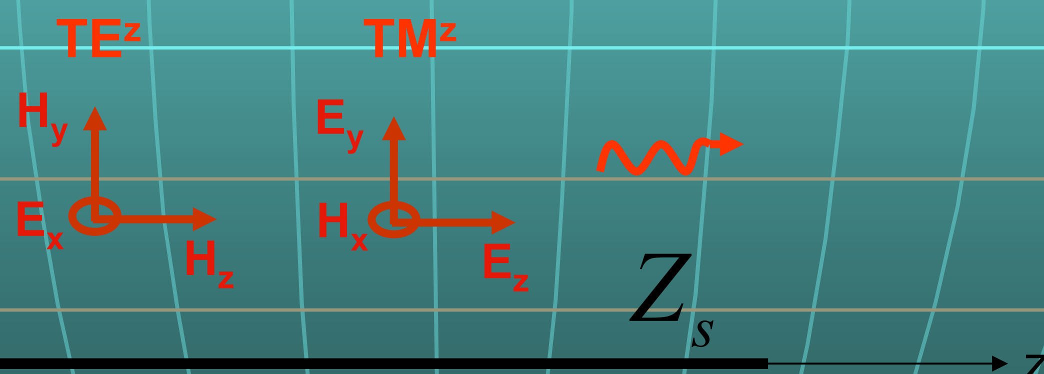
Model 1 – Impedance Surface

Transmission Line Model



$$Z_s = \frac{Z_g Z_d}{Z_g + Z_d}$$

Impedance Surface Model

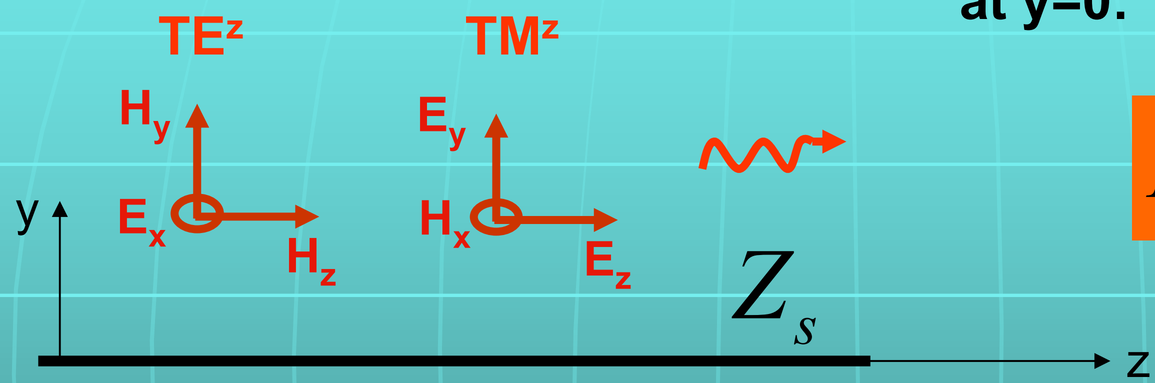


No fields beyond the impedance surface

S. A. Tretyakov, *Analytical Modeling in Applied Electromagnetics*, Boston, MA: Artech House, 2003

Model 1 – Impedance Surface

Impedance Surface Model



No fields beyond the impedance surface

Impedance Boundary Condition at y=0:

$$\vec{E} = Z_s \hat{y} \times \vec{H}$$

TE^z

$$E_x = Z_s^{TE} H_z \qquad E_x = E_0 e^{-jk_z^{TE} z - k_y^{TE} y}$$

$$k_y^{TE} = -\frac{j\omega\mu_0}{Z_s^{TE}} \qquad k_z^{TE} = k_0 \sqrt{1 - \left(\frac{\eta_0}{Z_s^{TE}}\right)^2}$$

TM^z

$$E_z = -Z_s^{TM} H_x \qquad H_x = H_0 e^{-jk_z^{TM} z - k_y^{TM} y}$$

$$k_y^{TM} = -j\omega\varepsilon_0 Z_s^{TM} \qquad k_z^{TM} = k_0 \sqrt{1 - \left(\frac{Z_s^{TM}}{\eta_0}\right)^2}$$

S. A. Tretyakov, *Analytical Modeling in Applied Electromagnetics*, Boston, MA: Artech House, 2003

Surface Waves

$$Z_s = \frac{Z_g Z_d}{Z_g + Z_d}$$

$$Z_d^{TE}(\omega,\theta) = \frac{j\eta_0}{\sqrt{\varepsilon_r - \sin^2\theta}} \tan(k_{zd} h)$$

$$Z_d^{TM}(\omega,\theta) = \frac{j\eta_0}{\sqrt{\varepsilon_r - \sin^2\theta}} \tan(k_{zd} h)\left(1 - \frac{\sin^2\theta}{\varepsilon_r}\right)$$

$$Z_g^{TE}(\omega,\theta) = Z_{g,L}^{TE}(\omega,\theta) + Z_{g,C}^{TE}(\omega,\theta)$$

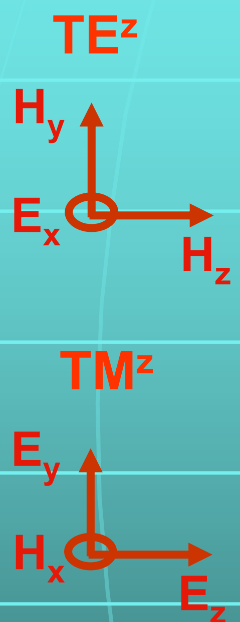
$$Z_g^{TM}(\omega,\theta) = Z_{g,L}^{TM}(\omega,\theta) + Z_{g,C}^{TM}(\omega,\theta)$$

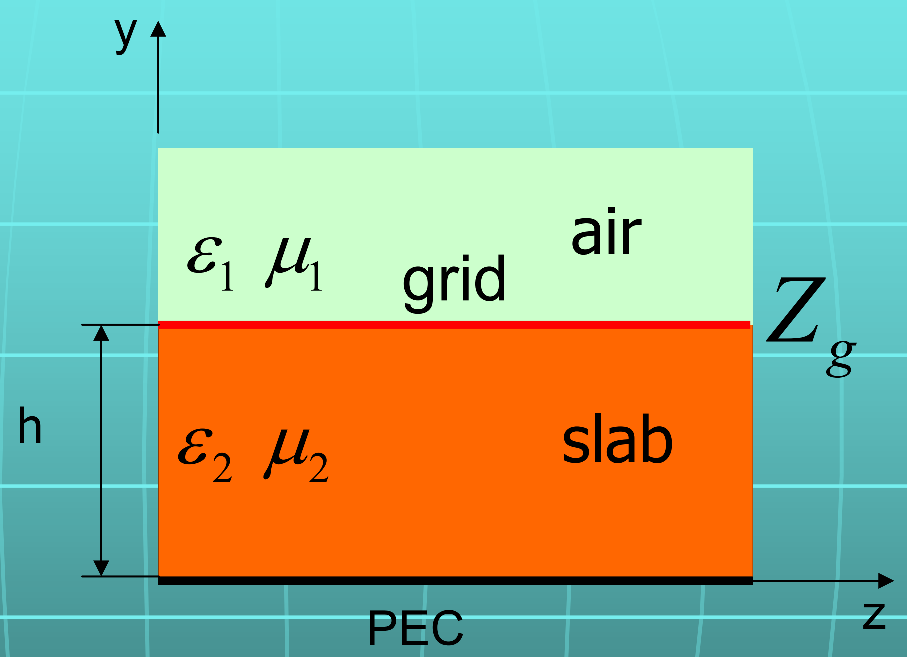
$$\sin(\theta) = \frac{k_z}{k_0} \Rightarrow Z_s \equiv Z_s(k_z)$$

$$k_z^{TE} = k_0\sqrt{1 - \left(\frac{\eta_0}{Z_s^{TE}(k_z^{TE})}\right)^2}$$

$$k_z^{TM} = k_0\sqrt{1 - \left(\frac{Z_s^{TM}(k_z^{TM})}{\eta_0}\right)^2}$$

Model 2 – Grounded Dielectric Slab with Grid Impedance on Air-Dielectric Interface





Two-sided impedance boundary condition at y=h

$$\vec{E}_1 = \vec{E}_2 = Z_g \hat{y} \times \left(\vec{H}_1 - \vec{H}_2 \right)$$

Dispersion Equations

Two-sided impedance boundary condition at y=h

TE^z-odd

$$E_{x1} = E_{x2} = Z_g^{TE}\left(H_{z1} - H_{z2}\right)$$

TM^z-even

$$E_{z1} = E_{z2} = -Z_g^{TM}\left(H_{x1} - H_{x2}\right)$$

Dispersion equations

$$\frac{\mu_2}{\mu_1}k_{y1} + k_{y2}\coth(k_{y2}h) = -\frac{j\omega\mu_2}{Z_g^{TE}}$$

$$k_{y2}\tanh(k_{y2}h) = -Z_g^{TM}\frac{j\omega\varepsilon_2 k_{y1}}{j\omega\varepsilon_1 Z_g^{TM} + k_{y1}}$$

Complex Wavenumber Plane

Branch points in the complex k_z-plane at $k_z = \pm k_1$

$\mathrm{Re}\{k_{y_1}\} > 0$ - proper modes on the top Riemann sheet

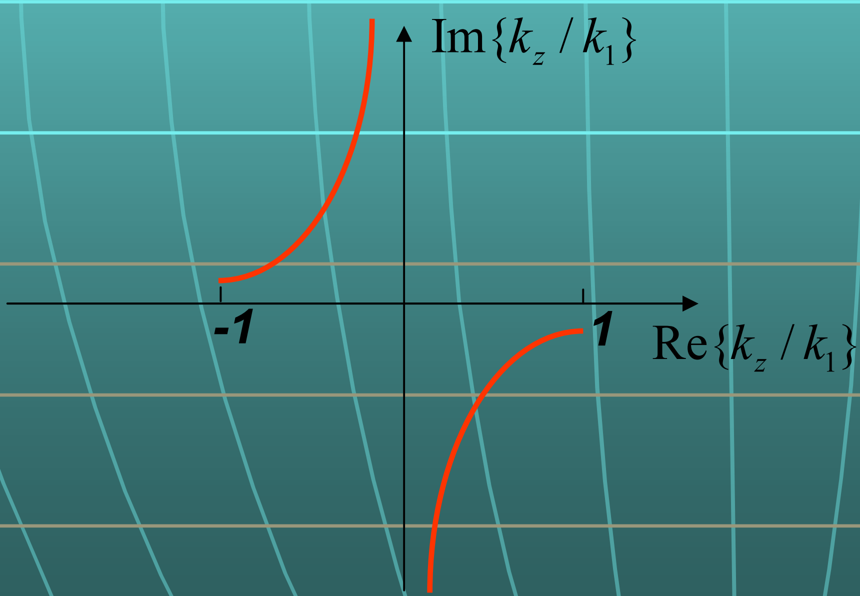
$\mathrm{Re}\{k_{y_1}\} < 0$ - improper modes on the bottom Riemann sheet

$\mathrm{Re}\{k_{y_1}\} = 0$ - branch cuts condition

Hyperbolic k_z-plane branch cuts:

$$\mathrm{Im}\{k_z\} = \frac{\mathrm{Im}\{k_1\}\,\mathrm{Re}\{k_1\}}{\mathrm{Re}\{k_z\}}$$

$$\left|\mathrm{Re}\{k_z\}\right| < \left|\mathrm{Re}\{k_1\}\right|$$

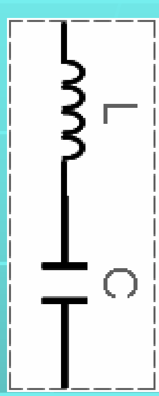


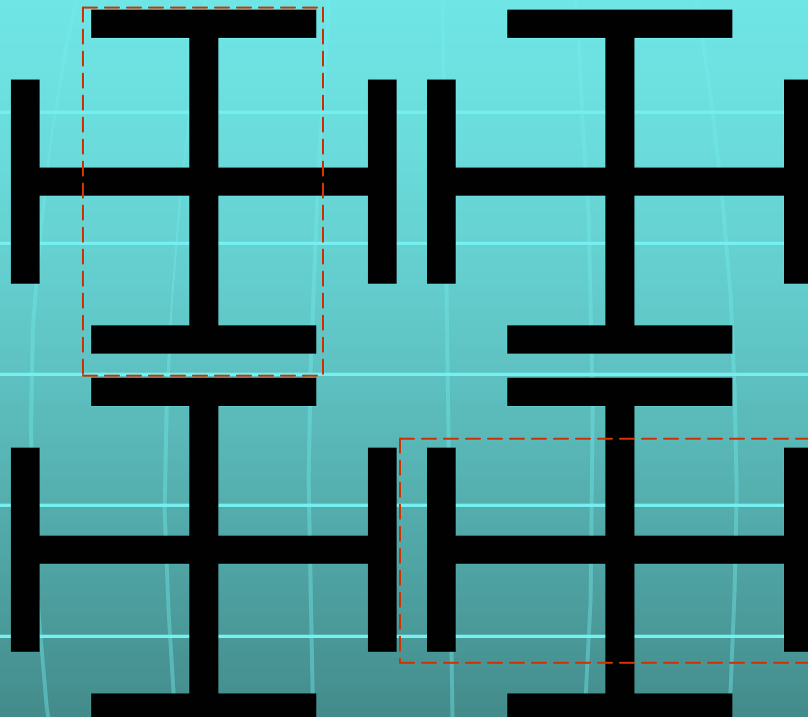
$$k_{y_i} = \sqrt{k_z^2 - k_i^2}$$

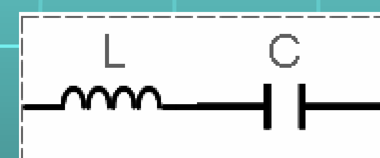
$$k_i^2 = n_i^2 \left(\frac{\omega}{c}\right)^2$$

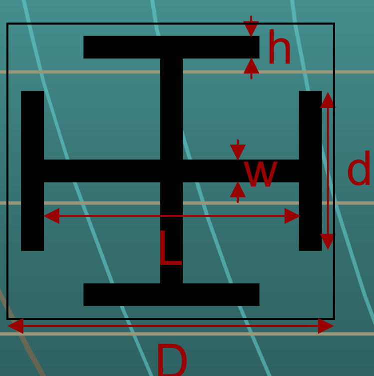
$$c = \left(\mu_0 \varepsilon_0\right)^{-1/2}$$

Jerusalem Cross Array





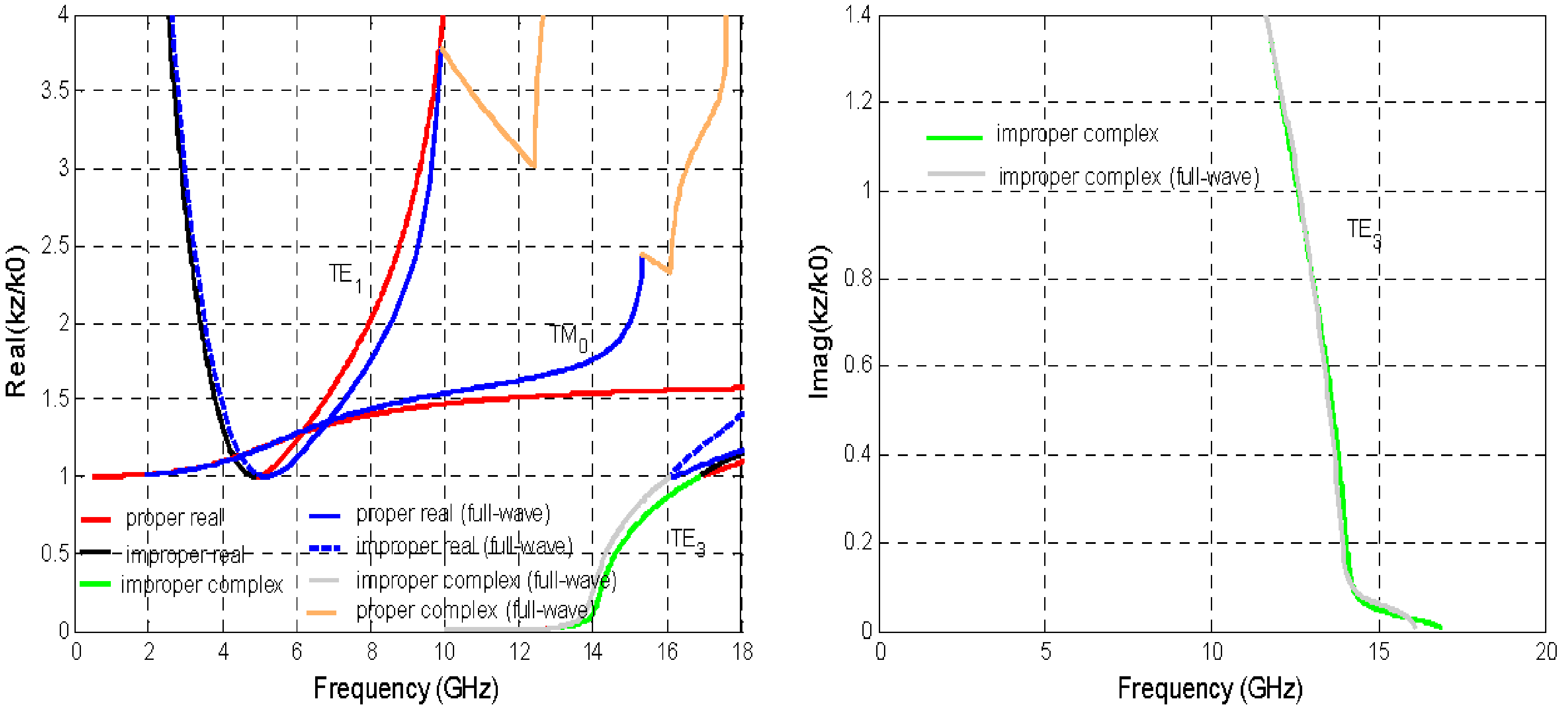




D = 4 mm, d = 2.8 mm,
h = w = 0.2 mm, L = 3.5 mm
substrate thickness is 6 mm
dielectric permittivity is 2.7

Dispersion Behavior of Surface Waves

Jerusalem cross HIS structure
Comparison with full-wave results

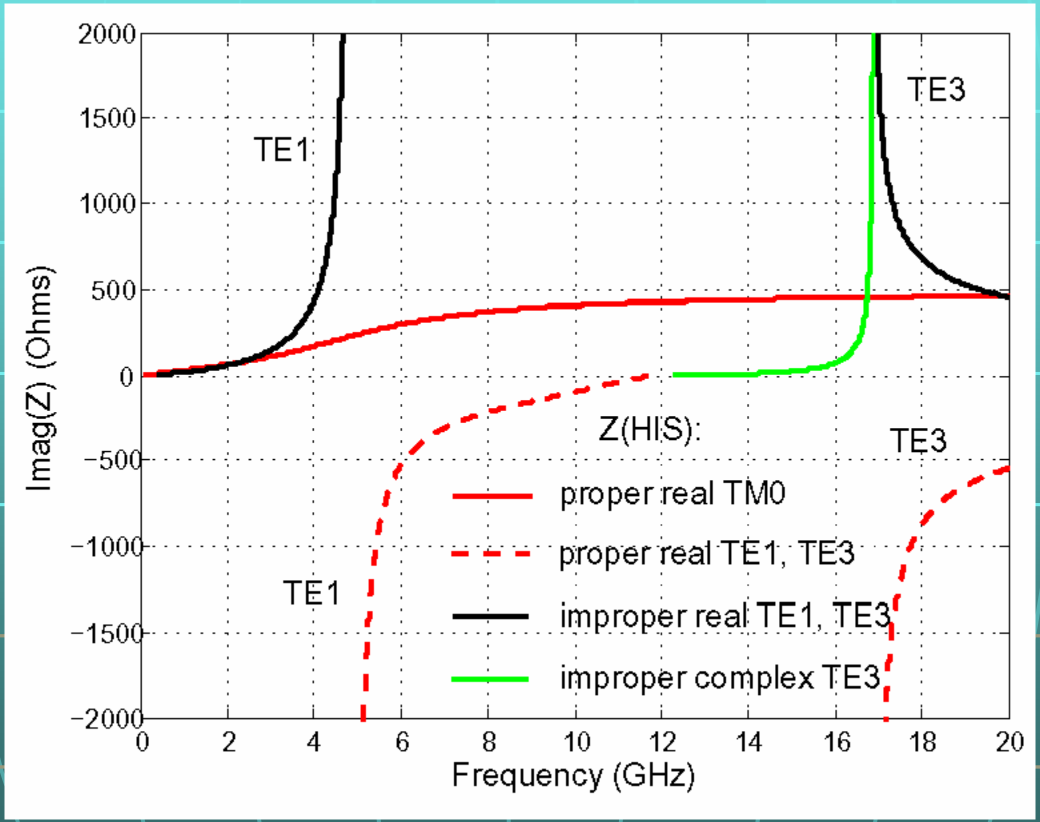


P. Baccarelli, S. Paulotto, and C. Di Nallo, "Full-wave analysis of bound and leaky modes propagating along 2D periodic printed structures with arbitrary metallization in the unit cell, IET Microwave Antennas Propagation, Vol. 1, No. 1, pp. 217-225, 2007.

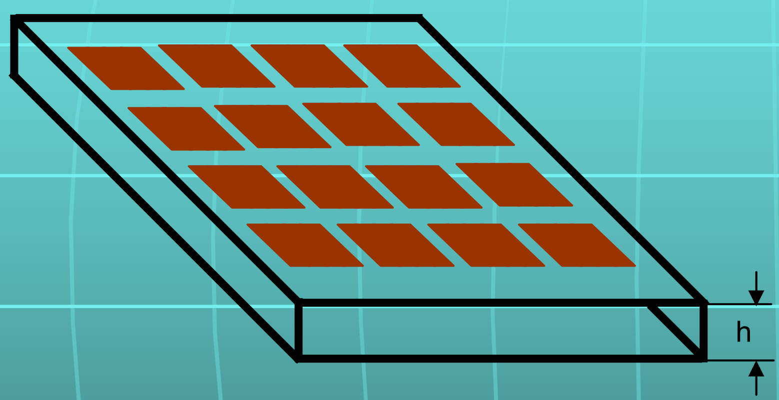
Surface Impedance of HIS

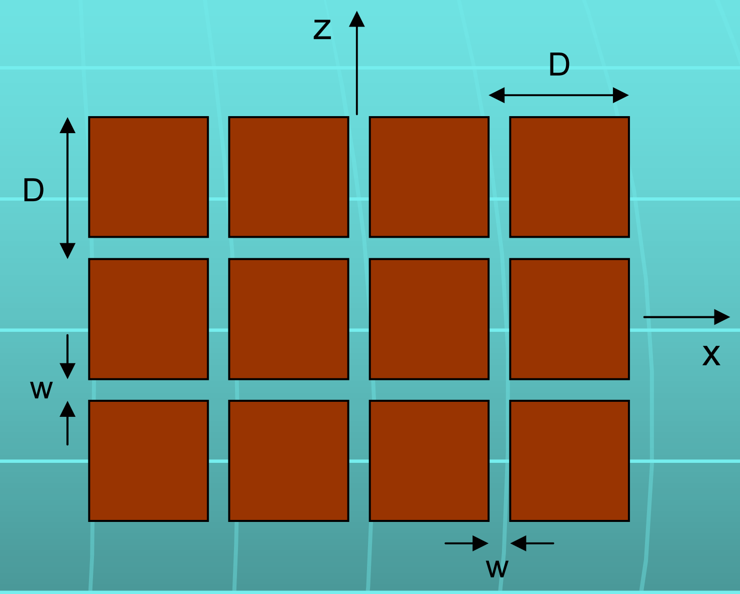
Jerusalem cross HIS structure

Surface impedance of HIS “seen” by surface waves



Patch Array

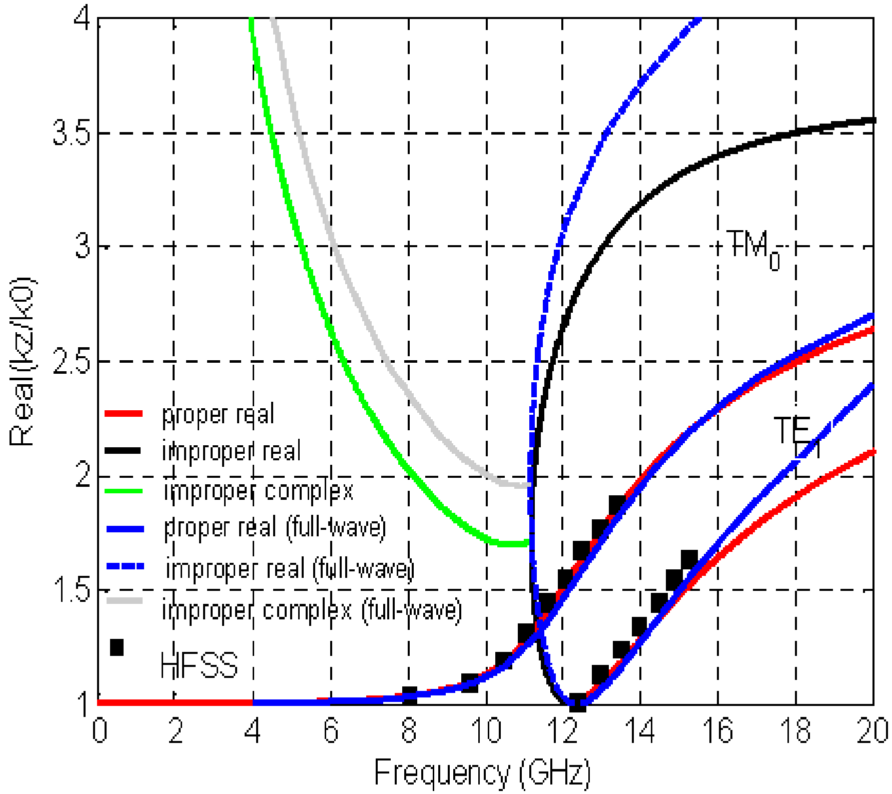


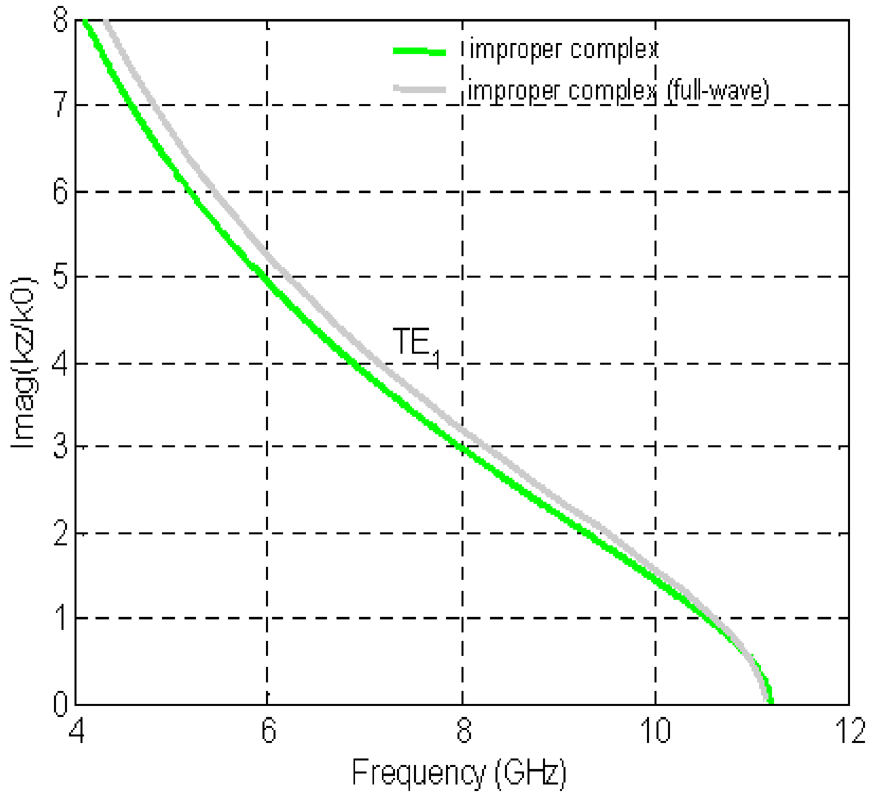


D = 2 mm, w = 0.2 mm
substrate thickness is 1 mm
dielectric permittivity is 10.2

Dispersion Behavior of Surface Waves

Patch HIS structure
Comparison with full-wave results



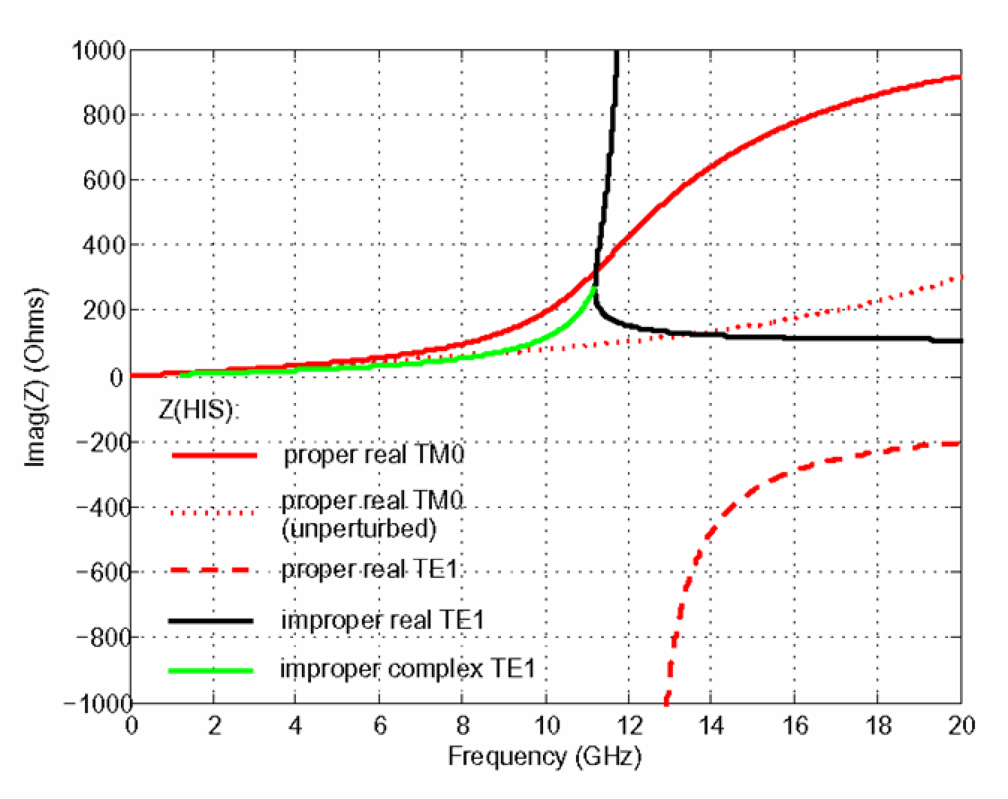


P. Baccarelli, S. Paulotto, and C. Di Nallo, "Full-wave analysis of bound and leaky modes propagating along 2D periodic printed structures with arbitrary metallization in the unit cell, IET Microwave Antennas Propagation, Vol. 1, No. 1, pp. 217-225, 2007.

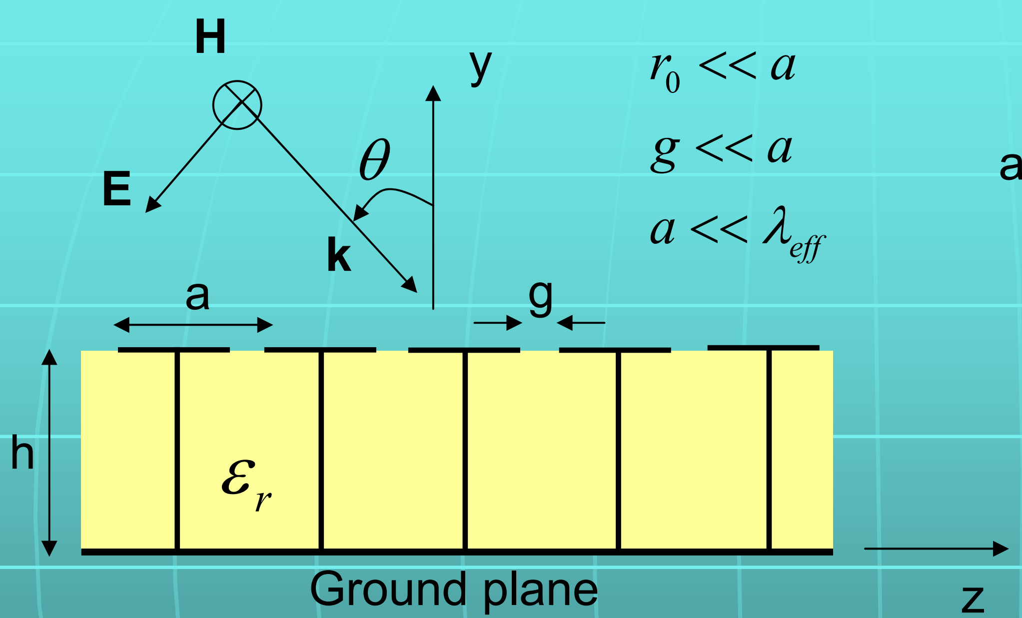
Surface Impedance of HIS

Patch HIS structure

Surface impedance of HIS “seen” by surface waves



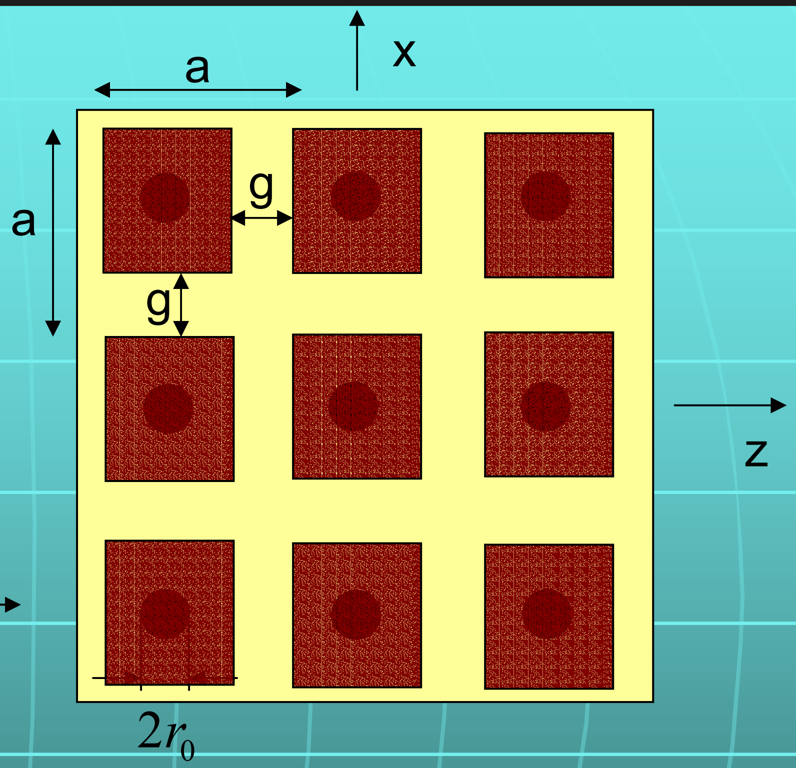
Mushroom Array



$r_0 << a$

$g << a$

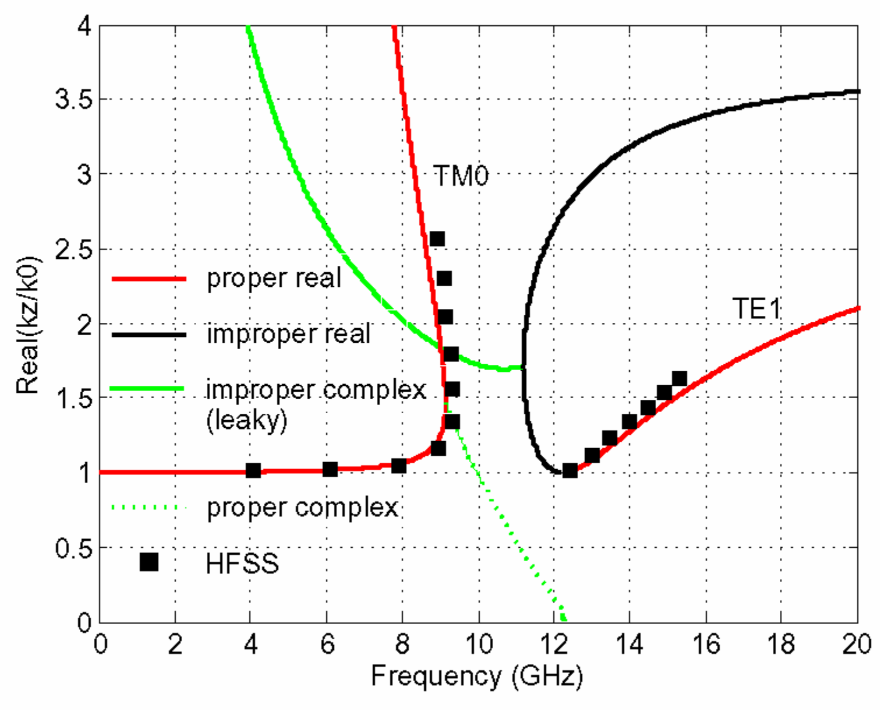
$a << \lambda_{eff}$

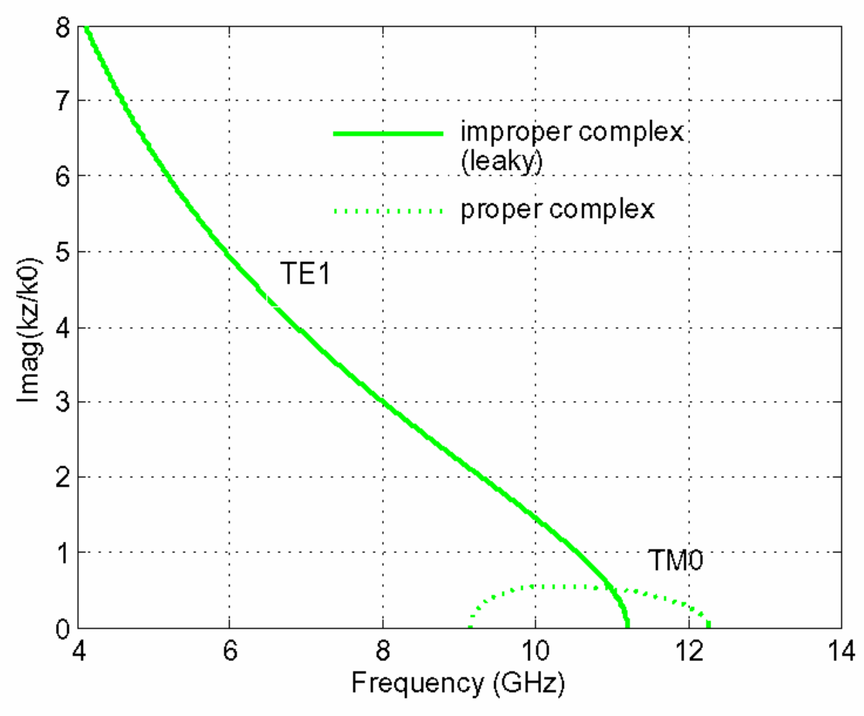


Period of vias is 2 mm
Period of patches is 2 mm
Gap is 0.2 mm
Radius of vias is 0.05 mm
Substrate thickness is 1 mm
Dielectric permittivity is 10.2

Dispersion Behavior of Surface Waves

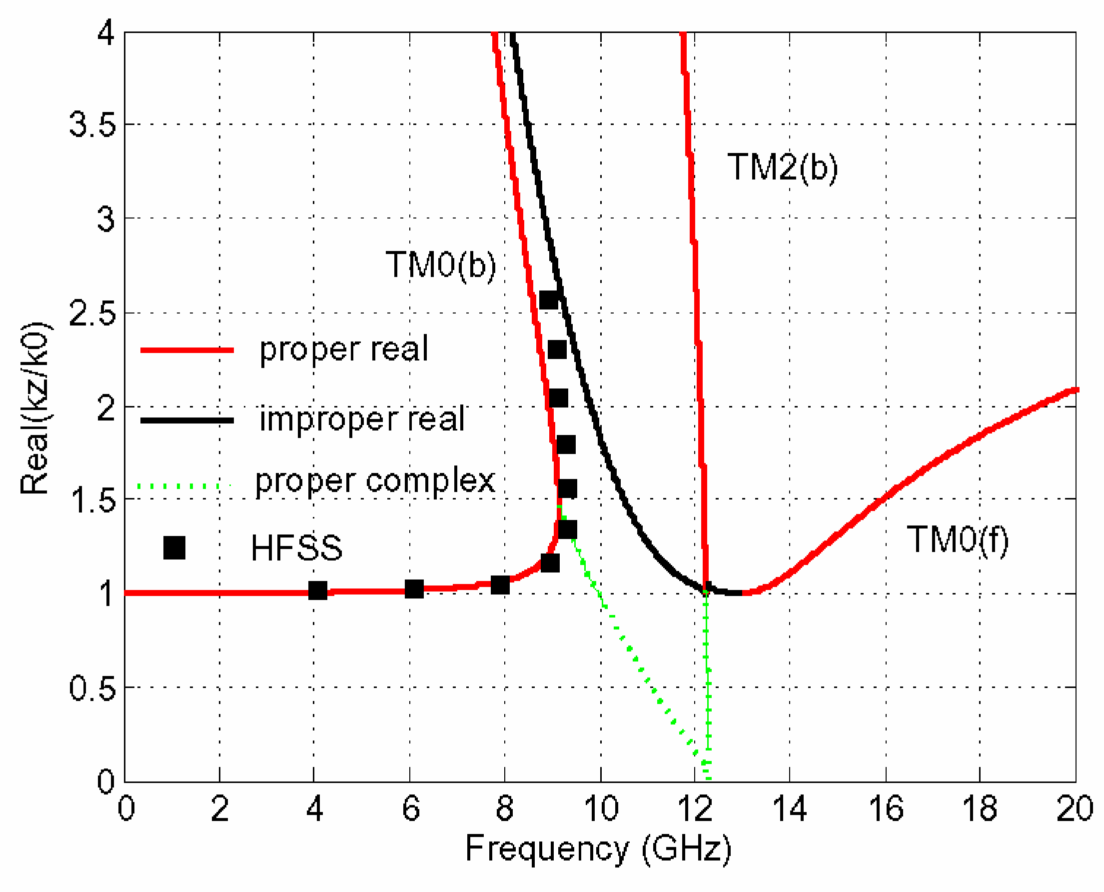
Mushroom HIS structure
Comparison with full-wave results





TM Surface Waves

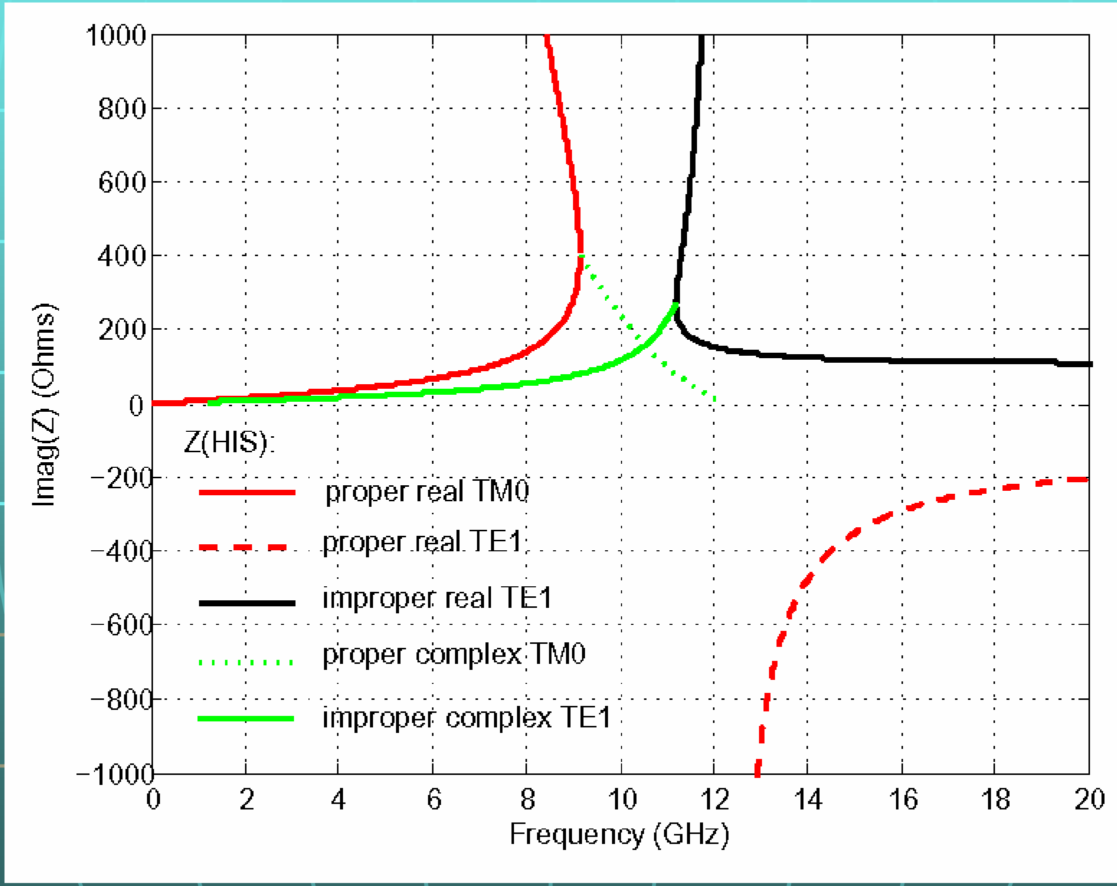
Mushroom HIS structure
Higher-order TM backward waves



Surface Impedance of HIS

Mushroom HIS structure

Surface impedance of HIS “seen” by surface waves



Conclusion

- **Accurate and rapid analysis of plane-wave interaction and surface-wave propagation on dense HIS structures**
- **Analytical modeling is based on the quasi-static approximation of full-wave scattering problem via the averaged impedance boundary condition. A homogenized surface grid impedance is expressed in terms of effective circuit parameters**
- **It is observed that in dense HIS structures no stopband between TE and TM surface-wave modes occurs at low frequencies. This is in contrast to conventional FSS structures, wherein stopbands occur due to Bragg's diffraction at resonance frequency**
- **Stopbands in mushroom HIS structures at low frequencies are due to backward waves associated with wire media slab**

Acknowledgment

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