Line-Source Excitation of Realistic Conformal Metasurface Cloaks

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Following our recently introduced analytical tools to model and design conformal mantle cloaks based on metasurfaces [Padooru et al., J. Appl. Phys., 112, 034907 (2012)], we investigate their performance and physical properties when excited by an electric line source placed in their close proximity. We consider metasurfaces formed by 2-D arrays of slotted (meshes and Jerusalem cross slots) and printed (patches and Jerusalem crosses) sub-wavelength elements. The electromagnetic scattering analysis is carried out using a rigorous analytical model, which utilizes the two-sided impedance boundary conditions at the interface of the sub-wavelength elements. It is shown that the homogenized grid-impedance expressions, originally derived for planar arrays of sub-wavelength elements and plane-wave excitation, may be successfully used to model and tailor the surface reactance of cylindrical conformal mantle cloaks illuminated by near-field sources. Our closed-form analytical results are in good agreement with full-wave numerical simulations, up to sub-wavelength distances from the metasurface, confirming that mantle cloaks may be very effective to suppress the scattering of moderately sized objects, independent of the type of excitation and point of observation. We also discuss the dual functionality of these metasurfaces to boost radiation efficiency and directivity from confined near-field sources.

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I. INTRODUCTION

In recent years, there has been an increased interest in the analysis and design of various electromagnetic invisibility cloaks due to their broad range of applications in camouflaging, non-invasive probing\(^1-^3\), low-interference communication\(^2\) and imaging\(^4\), among others. In particular, the coordinate transformation (CT) method, most commonly referred to as the transformation-optics method and first proposed by Pendry et al\(^4\) and Leonhardt\(^5\), has been successfully used to realize various electromagnetic cloaks with design frequencies ranging from microwaves\(^6-^9\) to visible light\(^10,^11\). Several other methods are also available to design invisibility cloaks, such as the scattering cancellation method (i.e., plasmonic cloaking\(^1-^3,^7,^9\)), anomalous localized resonance\(^12\), and transmission-line method\(^13\). All the above mentioned techniques are based on the exotic properties of bulk metamaterials, which are often challenging to realize even within today’s fabrication technology.

To overcome this issue, recently some of the authors have introduced a different cloaking technique\(^14-^16\) based on the concept of mantle cloaking or cloaking by a surface, that aims at reducing the overall visibility of objects of various shapes using a simple, infinitesimally thin conformal metasurface. By adjusting the design parameters of its inclusions, one can achieve the desired surface reactance, which is able to effectively cancel the main scattering orders from a given moderately sized object, therefore reducing its overall visibility. The inverse design of the metasurface inclusions for the given required surface reactance may still be a difficult process, since it typically involves a complex and time-consuming numerical optimization procedure. To overcome this problem, in Ref. 17, we have proposed a simple and accurate analytical model to design various metasurface cloaks, providing a clear analytical recipe for the collective response of individual elements that constitute metasurfaces conformal to cylindrical objects.

The mantle cloaking technique has been proven robust to frequency, losses and design variation, but so far the analysis has been limited to plane-wave scattering, i.e., the object and the cloak lie in the far-field of the source. In this work, we extend our rigorous analytical treatment of the mantle cloaking technique to the case of an electric line source placed in close proximity of a cloaked cylindrical object, in order to study the robustness of this technique to changes in excitation, in particular when sources are placed very near the object to be cloaked. We show that the design rules obtained in Ref. 17 for plane-wave incidence are still
very effective, even for near-field excitation, for both dielectric and conducting cylinders covered with slotted and printed metasurface mantle cloaks. We consider a variety of 2-D arrays with sub-wavelength periodic elements, such as mesh grids and patches\textsuperscript{18-23}, and Jerusalem crosses\textsuperscript{24,25}. The scattering problem is solved using the analytical eigenfunction-expansion method, which employs sheet impedance boundary conditions on the mantle cloak surface. Our analytical results are then accurately validated with full-wave simulations (HFSS\textsuperscript{26}).

A detailed study in regard to the applicability of the homogenized grid-impedance expressions, originally derived and validated for planar surfaces and plane-wave incidence (see for example, Refs. 19, 21–25), is presented for the case of an electric line source. Some considerations on the accuracy of these analytical expressions (which depend on the position of the line source and the observation points) are also provided, based on the comparison between analytical and full-wave numerical simulations. In general, very good agreement is found between analytical and numerical techniques, which expectedly deteriorate when sources or observation points are located at distances less than the period of the metasurface sub-wavelength inclusions.

In addition to confirming the robustness of the mantle cloaking technique, we discuss physical insights on the general scattering properties of cylindrical objects covered by metasurfaces near a line source. In this context, we also discuss how resonant metasurfaces may largely enhance the radiation of nearby electric line sources, based on Purcell effect, and can increase the directivity in the forward direction, compared to a bare metallic cylinder. These interesting properties may offer potential applications in low-profile conformal antennas.

The paper is organized as follows. In Section II, we present the formalism and analytical model for the analysis of 2-D dielectric and metallic cylindrical objects surrounded by metasurfaces (metameshes and metafilms), under an electric line source illumination. In Section III, we provide a detailed parametric study and demonstrate the robustness of conformal mantle cloaks. In Section IV, we discuss the accuracy and applicability of the analytical grid impedance expressions for cylindrical mantle cloaks for near-field excitation. Further, we show the effectiveness of the mantle cloaking technique for relatively larger dielectric objects in presence of a line source. In Section V, we demonstrate that patterned metasurfaces (basically printed patches) coating a metallic cylinder may also be used for low-profile conformal antenna applications, with improved directivity and enhanced radi-
ation. Finally, conclusions are drawn in Section VI. A time dependence $e^{j\omega t}$ is assumed throughout this study.

II. THEORETICAL ANALYSIS

FIG. 1. Schematic geometry of a dielectric cylinder covered by a conformal array of slotted Jerusalem crosses, illuminated by an electric line source carrying a current $I_0$: (a) 3-D view, (b) periodic grid of the planar metasurface, and (c) top view of (a).

Consider the geometry illustrated in Fig. 1(a), in which an infinite electric line source of constant current $I_0$ is positioned in the vicinity of an infinitely long dielectric cylinder with relative permittivity $\varepsilon_r$ and radius $a$, covered by a mantle cloak with radius $a_c$. The space be-
tween the cloak and the cylinder is filled by a dielectric with thickness $a_c-a$ and relative permittivity $\varepsilon_c$. The source and observation locations, along with the positions of the object and of the cloak, are shown in Fig. 1(c) in cylindrical $(\rho, \phi, z)$ and Cartesian $(x, y, z)$ coordinate systems, where the axis of the cylinder and that of the line source coincides with the $z$-axis. The coordinates of the line source are $(\rho_s, \phi_s)$, while the coordinates of the observation point are $(\rho, \phi)$. The distance between the electric line source and the cloak surface is denoted by $d$.

The relation between the variables in Cartesian coordinate system and those of the cylindrical system is given as

\[
R = \sqrt{\rho^2 + (\rho_s)^2 - 2\rho\rho_s \cos(\phi - \phi_s)}, \quad \rho_s = \sqrt{(x_s)^2 + (y_s)^2},
\]

and

\[
\rho = \sqrt{x^2 + y^2}.
\]

For a 2-D infinite electric line source modeled with current density

\[
J_e = \hat{\mathbf{z}} \frac{I_0}{\rho} \delta(\rho - \rho_s) = \hat{\mathbf{z}} \frac{I_0}{\rho} \delta(\rho - \rho_s) \delta(\phi - \phi_s),
\]

the incident field is given by

\[
E^i_z(\rho, \phi) = -\frac{k_0^2 I_0}{4\omega \varepsilon_0} H^{(2)}_0(k_0|\rho - \rho_s|),
\]

where $H^{(2)}_0(k_0|\rho - \rho_s|)$ is the Hankel function of the second kind. Employing the addition theorem for Hankel functions Eq. (2) can be written as

\[
E^i_z(\rho, \phi) = -\frac{k_0^2 I_0}{4\omega \varepsilon_0} \left\{ \sum_{n=-\infty}^{\infty} J_n(k_0\rho) H^{(2)}_n(k_0\rho_s) e^{jn(\phi - \phi_s)}, \quad \rho \leq \rho_s, \right. \\
\left. \sum_{n=-\infty}^{\infty} J_n(k_0\rho_s) H^{(2)}_n(k_0\rho) e^{jn(\phi - \phi_s)}, \quad \rho \geq \rho_s. \right. 
\]

In the presence of the cylinder, the scattered fields in each region are

\[
E^s_z(\rho, \phi) = -\frac{k_0^2 I_0}{4\omega \varepsilon_0} \left\{ \begin{array}{ll}
\sum_{n=-\infty}^{\infty} c_n H^{(2)}_n(k_0\rho) e^{jn\phi}, & \rho \geq a_c, \\
\sum_{n=-\infty}^{\infty} [a_n J_n(k_c\rho) + b_n Y_n(k_c\rho)] e^{jn\phi}, & a \leq \rho \leq a_c, \\
\sum_{n=-\infty}^{\infty} d_n J_n(k\rho) e^{jn\phi}, & \rho \leq a,
\end{array} \right.
\]

where $k_0 = \omega \sqrt{\mu_0 \varepsilon_0}$, $k_c = k_0 \sqrt{\varepsilon_c}$, and $k = k_0 \sqrt{\varepsilon_r}$ are the wave numbers in free-space, concentric dielectric region, and core regions, respectively. $J_n(\cdot)$ and $Y_n(\cdot)$ are the Bessel functions of the first and second kind. The unknown coefficients $a_n$, $b_n$, $c_n$, and $d_n$ are obtained by enforcing the following boundary conditions: (i) continuity of tangential electric
and magnetic fields at the core surface ($\rho = a$) and (ii) two-sided impedance boundary condition at the interface of the mantle cloak ($\rho = a_c$). The two-sided impedance boundary condition relates tangential electric fields to tangential magnetic fields, with the presence of sheet impedance:

$$\left. (E_z^i + E_z^s) \right|_{\rho = a_c^+} = \left. E_z^s \right|_{\rho = a_c^-} = Z_s \left[ \left. (H_\phi^i + H_\phi^s) \right|_{\rho = a_c^+} - \left. H_\phi^s \right|_{\rho = a_c^-} \right],$$  \hspace{1cm} (5)$$

where $Z_s$ is the surface impedance of the mantle cloak covering the cylinder. $H_\phi^i$ and $H_\phi^s$ are the corresponding magnetic field components, obtained using Maxwell’s Faraday equation. The mantle cloaks considered in the following are realized using common metasurface elements [with a typical geometry shown in Fig. 1(a)]. With the assumption that the inclusions are deeply sub-wavelength, the metasurface cloak can be characterized by a homogenized surface with average impedance $Z_s = R_s + jX_s^{18,19}$, where $R_s$ is the surface resistance associated with losses and $X_s$ is the surface reactance, accounting for the stored energy, which can be either inductive ($X_s > 0$) or capacitive ($X_s < 0$) depending on the metasurface structure. For simplicity, here we assume a lossless surface with $R_s = 0$, as mantle cloaking has been shown to be very robust to moderate absorption$^{15}$. The limiting case $|X_s| \to \infty$ corresponds to a bare dielectric cylinder with no cloak, as the metasurface has no interaction with the impinging field. Since, closed-form analytical expressions for the surface impedance of cylindrical metasurfaces are difficult to obtain, and since the sub-wavelength size of the inclusions ensures that the local curvature of the surface may be considered large, as successfully done in Ref. 17 we use available analytical expressions for planar metasurfaces to model the mantle cloaks. Details of these analytical expressions for various planar periodic sub-wavelength elements (i.e. meshes, patches, Jerusalem crosses, and slotted Jerusalem crosses), are provided in the Appendix of Ref. 17. In the following, we explore the cloak operation and validity of these approximate expressions for excitation using nearby electric sources. We show that, under certain conditions with respect to the location of the source, it is safe to ignore the evanescent fields scattered by the grid on the observation plane and it is still possible to safely use design formulas originally derived for planar metasurfaces or FSS arrays excited by plane waves.
III. NUMERICAL RESULTS

In this section we study the robustness of the mantle cloak design for an electric line source positioned in the vicinity of the object, using the analytical model described in Section II. Analogous to the analysis carried out in Ref. 17 for plane-wave excitation (i.e., for a source situated at infinity), in order to synthesize the required surface reactance we use the analytical grid-impedance expressions developed in Refs. 18, 19, 24, and 25 for a planar surface. Our analytical results, and the effectiveness of this approximation, are validated using full-wave numerical simulations.

![Analytical calculations of the distribution of the magnitude of $E_z$-field on the $\phi$-plane](image)

(a) in free space, (b) without the cloak, and (c) with the mesh-grid cloak. The red colored dot in (a), (b), and (c) indicates the position of the line source, the white circle in (b) shows the boundaries of the cylinder without the cloak, and the red dashed circle in (c) shows the presence of the mesh-grid cloak around the cylinder. The position of the line source from the cloak interface is $d = 0.1\lambda_0$.

FIG. 2. Analytical calculations of the distribution of the magnitude of $E_z$-field on the $\phi$-plane: (a) in free space, (b) without the cloak, and (c) with the mesh-grid cloak. The red colored dot in (a), (b), and (c) indicates the position of the line source, the white circle in (b) shows the boundaries of the cylinder without the cloak, and the red dashed circle in (c) shows the presence of the mesh-grid cloak around the cylinder. The position of the line source from the cloak interface is $d = 0.1\lambda_0$. 
A. Dielectric cylinder

For all the configurations studied in this paper, without lack of generality the position of the line source is varied only along the $x$-axis (no azimuthal variation, $\phi_s = 0$, $y_s = 0$, $\rho_s = x_s$). The location of the line source is therefore simply indicated by the distance $d$ between the surface of cloak and the line source, as shown in Fig. 1(c). We first consider an infinite electric line source located at the point $S(\rho_s, \phi_s) = (0.2\lambda_0, 0)$, where $\lambda_0$ is the wavelength of operation and a current amplitude $I_0 = 1\, A$. The source excites a 2-D infinitely long dielectric cylinder with $\varepsilon_r = 10$ and radius $a = \lambda_0/10$ placed at the origin, implying that $d = 0.1\lambda_0$. The electric field distribution $E_z$, calculated using the analytical model given in Section II, is shown in Fig. 2, with $E_z$ being the instantaneous field in each region. The fields are calculated on the $x$-$y$ or $\phi$-plane, where dimensions along each coordinate axis are normalized to $\lambda_0$. Obviously, the dielectric cylinder scatters the incident field in different directions and perturbs the impinging field distribution. Following our previous analysis for plane-wave excitation\textsuperscript{15,17}, we know that, by cloaking the cylinder with an inductive mesh-grid, we may drastically suppress its scattering. Using the results of Section II, the required surface reactance at the design wavelength corresponds to 49.8\,\Omega. We designed a conformal mesh grid, as shown in Fig. 3, to synthesize this average surface reactance using the formulas available in Ref. 17. The important design parameters are $D = \lambda_0/16$, $w = \lambda_0/200$, $\varepsilon_r = 10$, and $a_c = a$. The corresponding electric field distribution $E_z$ calculated using the analytical model (i.e., using the average surface impedance) is shown in Fig. 2(c), and can be compared with the uncloaked cylinder [Fig. 2(b)] and the radiation of the line current in free-space [Fig. 2(a)]. Evidently, the mantle cloak is able to significantly suppress the overall scattering and restore the original, azimuthally symmetric field distribution from the source, as if the cloaked object is effectively invisible. To validate these results, we show in Fig. 4 the corresponding full-wave numerical simulations considering the realistic mesh geometry for the two cases of Fig. 2. A computational domain similar to the analytical model was considered in our simulations, with boundaries terminated by perfectly matched layers (PMLs). The full-wave simulations indeed validate the concept of mantle cloaking and ensure that, even for objects placed in the very close neighborhood of a line source, (i) the cloaking effect is preserved, (ii) the analytical formulas employed for the design, and originally developed for a planar metasurface excited by a plane wave, still hold. In order
to gain physical insights into the transparency effect, in Fig. 5 we show the variation of the electric field $E_z$ along the $y$-axis for three different cases: free space (no cylinder near the source), bare dielectric cylinder and cloaked dielectric cylinder. Indeed, the field values for the cloaked cylinder have comparable amplitudes outside the region of the cloak as if the cylinder were not present at all. Obviously, a bare cylinder scatters fields in all directions and, hence, lower field amplitude is observed (red line). The accuracy of our analytical design formulas are also demonstrated in Fig. 5 by comparing the analytical results based on a homogenized surface model with full-wave results. It is clear that the analytical model can fully capture the physics of this scattering problem, confirming its applicability to a curved surface and a near-field excitation. The inevitable small discrepancies close to the grid surface are obviously associated with the higher-order evanescent fields neglected in the analytical model. Still their influence is limited to a region of space very close to the patterned surface and it does not influence the overall cloak performance.

In the next example, we consider the same dielectric cylinder and line source excitation, but a different mantle cloak design. Here we use a slotted Jerusalem-cross metasurface to realize the inductive metasurface cloak, whose geometry is shown in Fig. 1. The parameters of the optimized cloak are again obtained from the formulas given in the Appendix of Ref. 17, originally derived for plane-wave incidence: $D = \lambda_0/15$, $w = \lambda_0/200$, $g = \lambda_0/650$, $t = \lambda_0/22$, and $a_c = 1.05a$. The radius of the cloak in this case is slightly larger than in the previous...
FIG. 4. Full-wave simulation for the magnitude of the $E_z$-field distributions on the $\phi$-plane: (a) in free space, (b) without the cloak, and (c) with the mesh-grid cloak. The black dot in (a), (b), and (c) indicates the position of the line source, the white circle in (b) shows the boundaries of the cylinder without the cloak, and the red dashed circle in (c) shows the mesh-grid cloak around the cylinder. The distance of the line source from the cloak interface is $d = 0.1\lambda_0$.

case, where it was conformal to the dielectric cylinder, as required to tune the required value of $X_s$ at the design frequency. The medium between the dielectric and the slotted Jerusalem cross cloak is assumed to be free-space. Figure 6, similar to Fig. 5, shows the field distribution on a transverse line crossing the cylinder. Also in this case, the field amplitudes are fully restored by the cloak to levels comparable to the case of free-space radiation. The field distributions (color plots as those shown in Figs. 2 and 4) for analytical and full-wave simulations, not reported here for sake of brevity, confirm also in this case an excellent agreement.
FIG. 5. Electric field distributions $E_z$ along the $y$-axis: free space, dielectric cylinder without cloak, and dielectric cylinder covered with mesh-grid cloak. The vertical dashed bold lines represent the position of the mesh-grid cloak, and the shaded region represents the dielectric cylinder. The distance of the line source from the cloak interface is $d = 0.1\lambda_0$. The analytical results are represented by solid and dash-dot lines and full-wave simulation results by symbols (crosses and plus signs).

FIG. 6. Similar to Fig. 5, but for the slotted Jerusalem-crosses cloak, with $d = 0.1\lambda_0$. 
B. Conducting Cylinder

In this section, we extend the cloak design to the challenging case of a conducting cylinder with same dimensions, for which a capacitive surface reactance is required for good scattering suppression. To realize capacitive values of reactance, we use inclusions with complementary geometries compared to those studied in Section III A, i.e., 2-D arrays of printed patches and Jerusalem crosses, as shown in Fig. 7. In this case the field cannot penetrate the object and, due to the zero boundary impedance, we must consider a gap between the cloak and the object to avoid an electric short.

FIG. 7. Schematic geometries of conducting cylinders covered by metasurface mantle cloaks: (a) patch array cylindrical cloak, and (b) Jerusalem-cross cylindrical cloak.

We use the same design parameters for the patch-array cloak and the conducting object considered in Ref. 17, designed using the analytical expression for the grid-impedance given in the Appendix of Ref. 17, which were originally derived for plane-wave incidence. The cloak parameters and the line source position are summarized in Table I, where $N$ is the number of unit-cells surrounding the cylinder. In Fig. 8, similar to Figs. 2 and 4, we show the electric field $E_z$ distributions with and without cloak using the homogenized analytical model and full-wave simulations. Also in this case, the two sets of results agree very
well, and confirm that the scattering is greatly suppressed also for this near-field excitation, making the conducting object largely less visible. We have obtained analogous results using Jerusalem cross metasurfaces, not shown here for sake of brevity. Figures 9 and 10 are similar to Fig. 6, but for a conducting cylinder covered with the patch metasurface and with Jerusalem-cross metasurface, respectively. The field values for the cloaked conducting cylinder are almost identical to free-space radiation, in particular outside the cloak region. Further, the figures show good agreement between our analytical results and full-wave simulations, confirming that the analytical grid-impedance expressions derived for the planar sub-wavelength metasurface can still be used to tailor the surface reactance of cylindrical mantle cloaks, even when the covered object is conducting and is placed very close to the exciting source.

### TABLE I. Parameters of the 2-D sub-wavelength metasurface mantle cloaks conformal to a PEC cylinder, with $a = \lambda_0/10$.  

<table>
<thead>
<tr>
<th>Metasurface type</th>
<th>N</th>
<th>$D$</th>
<th>$w$</th>
<th>$g$</th>
<th>$t$</th>
<th>$\varepsilon_c$</th>
<th>$a_c = a$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patches</td>
<td>10</td>
<td>$\lambda_0/14$</td>
<td>-</td>
<td>$\lambda_0/222$</td>
<td>-</td>
<td>25</td>
<td>1.15$a$</td>
<td>0.1$\lambda_0$</td>
</tr>
<tr>
<td>JC</td>
<td>6</td>
<td>$\lambda_0/8$</td>
<td>$\lambda_0/125$</td>
<td>$\lambda_0/111$</td>
<td>$\lambda_0/28$</td>
<td>13</td>
<td>1.15$a$</td>
<td>0.15$\lambda_0$</td>
</tr>
</tbody>
</table>

### C. Dependence of the Mantle Cloak Operation with the Line Source Position

In the previous sections, the robustness and effectiveness of several mantle cloak designs has been assessed for dielectric and conducting cylindrical objects using metasurfaces with various printed and slotted periodic elements. Here we analyze the dependence of the cloaking effect with the line source position. This becomes particularly important when the distance between the cloaked object and the line source is comparable with the metasurface granularity. We first consider a dielectric cylinder covered by a mesh-grid cloak, as the case studied in Section III A. For the same cloaked dielectric object in Fig. 3, we show in Fig. 11 the electric field distributions, obtained analytically, for different line-source positions: $d = 0.015\lambda_0$, $0.1\lambda_0$, and $0.4\lambda_0$; the corresponding results for no cloak cases are also presented for a fair comparison. It is seen that the same conformal mesh-grid metasurface can effectively cloak the dielectric cylinder independent of the position of the line.
FIG. 8. $E_z$-field distributions on the $\phi$-plane, calculated using the analytical model: (a) with the patch-array cloak and (b) without the cloak; calculated using the full-wave simulations: (c) with the patch-array cloak and (d) without the cloak. The black and red dots in (a), (b), (c), and (d) indicate the position of the line source, the white circles in (b) and (d) show the boundaries of the PEC cylinder without the cloak, and the red dashed circles in (a) and (c) show the presence of the patch array cloak around the PEC cylinder. The distance of the line source from the cloak interface is $d = 0.1\lambda_0$.

source, even in the extreme case in which it is located in very close proximity ($d = 0.015\lambda_0$) to the cloaked object. Even more importantly, we have verified that very similar cloaking conditions are obtained using full-wave numerical simulations (not shown here for sake of brevity), ensuring that the cloak design is very robust to the location of the excitation even in the realistic case of a patterned surface. We discuss in the following section the validity
FIG. 9. Electric field distributions $E_z$ along the $y$-axis: free space, conducting cylinder without cloak, and conducting cylinder covered with patch-array cloak. The vertical dashed bold lines represent the position of the cloak, and the shaded region represents the conducting cylinder. The distance of the line source from the cloak interface is $d = 0.1\lambda_0$. The analytical results are represented by solid and dash-dot lines and full-wave simulation results by symbols (crosses and plus signs).

FIG. 10. Similar to Fig. 9, but for a Jerusalem-cross cloak, with $d = 0.15\lambda_0$.

... of the analytical model when the excitation gets too close to the patterned surface.
FIG. 11. Electric field distributions $E_z$ obtained using the analytical model for $d = 0.015\lambda_0$: (a) with and (b) without the mesh-grid cloak; for $d = 0.1\lambda_0$: (c) with and (d) without the mesh-grid cloak; for $d = 0.4\lambda_0$: (e) with and (f) without the mesh-grid cloak. The red colored dots indicate the position of the line source, the white circles in (b), (d), and (f) show the boundaries of the dielectric cylinder without the cloak, and the red dashed circles in (a) and (c), and black dashed circle in (e) show the presence of the mesh-grid cloak around the dielectric cylinder.

IV. LIMITATIONS AND ACCURACY OF THE ANALYTICAL MODEL

In this section we discuss in more detail the applicability of the analytical model for the surface impedance employed in the previous section when the excitation gets too close to the surface. The analytical grid-impedance expressions used in this work, were originally derived and validated for planar surfaces and plane-wave incidence$^{18,19,21-25}$. In Ref. 17, we have successfully shown and verified the accuracy of these expressions for tailoring the required surface reactance of cylindrical metasurfaces, under far-field plane-wave illumination. Recently in Ref. 29, some of the authors have studied the scattering problem of various
planar periodic elements on grounded dielectric slabs, excited by an aperiodic line source, wherein analytical grid-impedance expressions$^{18,19,24,25}$ have been used to characterize the corresponding grid impedance. In Ref. 29, a detailed study of the dependence on the source position, observation points and the periodicity of the sub-wavelength elements has been carried out and some important conclusions with respect to the accuracy of the analytical grid-impedance expressions were drawn. Hence, there is a necessity for validating these expressions in the scenario considered in the present paper, especially when the surface is illuminated in close proximity.

We first consider the dielectric cylinder covered by the conformal mesh-grid considered in Fig. 3. Figures 12, 13, and 14 show the variation of $E_z$ versus observation angle $\phi$ for different positions of the line source $d$ at various observation points $\rho$ (with $b$ being the distance between the surface of the cloak and the observation point, as shown in the insets of Figs. 12, 13, and 14). We compare analytical results obtained using the average surface impedance and full-wave numerical simulations. Figure 12 shows the variation of $E_z$ versus observation angle $\phi$ for an electric line source located at $d = 0.015\lambda_0$, in very close proximity of the cloaked object, for different observation points: $b = 0.002\lambda_0$, $b = 0.1\lambda_0$, and $b = 0.4\lambda_0$. It can be observed that, the results obtained by the analytical model show quite strong disagreement with the numerical results, even for observation points rather distant from the surface. This is because the cylindrical cloak lies very close to the line source, and the approximation of the homogenized grid-impedance expressions does not hold. The analytical expressions are valid provided that:

1. The incident field produced by the line source, which is independent of the $z$-coordinate, varies very little over the unit-cell of the metasurface in the $\phi$-direction and the evanescent fields, which decay exponentially from the source, have negligible amplitudes at the cloak interface. Following the discussion in Ref. 29, and the study carried out in this work, it is noticed that these conditions are satisfied provided the position of the line source $d$ from the cloak is such that $d \geq D$, where $D$ is the period of the sub-wavelength elements forming the metasurface.

2. The evanescent fields scattered by the cloak associated to higher-order Floquet harmonics have negligible amplitude at the observation point. Provided that condition 1 is satisfied, the periodic surface can be described using a Floquet expansion, for which
FIG. 12. Electric field distributions around a mesh-grid covered dielectric cylinder illuminated by a line source located at $d = 0.015\lambda_0$, as a function of azimuthal angle ($\phi$) at design frequency $f_0$ for different observation positions: (a) $b = 0.002\lambda_0$, (b) $b = 0.1\lambda_0$, and (c) $b = 0.4\lambda_0$. The analytical results are represented by solid lines and full-wave simulation results by dashed lines.

only the dominant term is propagating. The higher-order scattered harmonics become very small at distances on the order of the grid period $D$. Hence, for this condition to be satisfied, the observation point $b$ should be such that $b \geq D$.

It should be noted that the above two conditions are not necessarily met for all the grids considered in the previous section (for example the slotted Jerusalem cross and the patch array grids). Even if for the observation point located in close proximity to the surface $b = 0.002\lambda_0$, a disagreement is reasonably expected, it is interesting to notice that even for observation points far away $b \geq D$ (where $b = 0.1\lambda_0$, $b = 0.4\lambda_0$, and $D = \lambda_0/16 = 0.0625\lambda_0$),
FIG. 13. Similar to Fig. 12, but for a line source located at $d = 0.1\lambda_0$, with different observation positions: $b = 0.01\lambda_0$, $b = 0.2\lambda_0$, and $b = 0.5\lambda_0$.

FIG. 14. Similar to Fig. 12, but for a line source located at $d = 0.4\lambda_0$, with different observation positions: $b = 0.01\lambda_0$, $b = 0.2\lambda_0$, and $b = 0.7\lambda_0$.

the analytical results shown in Figs. 12(b) and 12(c) are still not accurate. This is due to the fact that the limitation on $d$ is not satisfied. In spite of the quantitative discrepancies in the prediction of the exact field behavior, one can still notice that the cloaking effect is confirmed, as shown in Fig. 11(a). This is quite remarkable and is a further highlight of the robustness of the mantle cloaking technique.
To further investigate the applicability of the analytical model, we consider now the cases when \( d \geq D \): 1) \( d = 0.1\lambda_0 \) and 2) \( d = 0.4\lambda_0 \), for observation points: \( b = 0.01\lambda_0, \ b = 0.2\lambda_0, \ b = 0.5\lambda_0, \) and \( b = 0.7\lambda_0 \). Specifically, we study the variation of \( E_z \) as a function of \( \phi \). The analytical results are reported in Figs. 13 and 14 along with full-wave numerical comparisons. It can be observed that in both cases \( d = 0.1\lambda_0 \) and \( d = 0.4\lambda_0 \) excellent agreement between analytical and full-wave simulations are obtained for \( b = 0.2\lambda_0, b = 0.5\lambda_0, \) and \( b = 0.7\lambda_0, \) consistent with Figs. 5 and 6. This is because the evanescent fields produced by the line source at the cloak interface are negligible, and the amplitude of the evanescent fields scattered by the grid are negligible at the observation points. For observation points very close to the cloak \( b = 0.01\lambda_0, \) strong disagreement between analytical and full-wave results is found for different values of \( d \). This is expected when taking into account that the evanescent Floquet harmonics scattered by the grid are significant at the observation point and cannot be neglected. Hence, the true field (microscopic field) calculated by the numerical solver shows oscillations in the field behavior, while the averaged field (macroscopic field averaged over one unit-cell) is well represented by the analytical model (particularly, in Fig. 13). In Figs. 15, 16, and 17 a similar analysis has been carried out for the other patterned cloaks considered above (slotted Jerusalem crosses, patch arrays, and Jerusalem crosses). For sake of brevity, here results are shown only for one specific location of the line source, i.e., \( d = 0.1\lambda_0 \) for slotted Jerusalem crosses and patch arrays, and \( d = 0.15\lambda_0 \) for Jerusalem crosses, such that \( d \geq D \), where \( D \) is period of the corresponding sub-wavelength elements, i.e., \( D \approx 0.06\lambda_0 \) for slotted Jerusalem crosses, \( D \approx 0.071\lambda_0 \) for patch arrays, and \( D \approx 0.125\lambda_0 \) for Jerusalem crosses. The cloak parameters and the objects are similar to those considered in Section III, and the field behaviors are calculated at different observation points away from the cloaks and compared with the full-wave numerical simulations. For observation points \( b \) very close to the cloak such that \( b < D \), the field behavior predicted by the analytical model is not accurate when compared to the numerical results (the results are not included here for sake of brevity). However, when \( b \) becomes comparable to period of the grid \( D \) (for the case of slotted Jerusalem crosses: \( b = 0.05\lambda_0 \) and \( D \approx 0.06\lambda_0 \), shown in Fig. 15, and for the patch arrays: \( b = 0.05\lambda_0 \) and \( D \approx 0.071\lambda_0 \), shown in Fig. 16), the analytical results agree better with the numerical results, and are even more accurate as \( b \) increases, i.e., for \( b = 0.2\lambda_0 \) and \( 0.5\lambda_0 \), shown in Figs. 15 and 16. For the Jerusalem cross cloak shown in Fig. 17, it can be observed that for \( b < D \) (\( b = 0.05\lambda_0 \) and \( D = 0.125\lambda_0 \)),
the analytical result shows some discrepancies. This is again due to the fact that the evanescent field scattered by the cloak is not negligible at this observation position. For $b = 0.2\lambda_0$ and $0.7\lambda_0$, the agreement between the analytical result and full-wave results improves. Finally, it is interesting to point out that for the high-impedance surfaces (HIS) analyzed in Ref. 29, the homogenized grid-impedance expressions were shown to be valid provided that the thickness of the grounded dielectric slab is comparable to the period $D$ of the sub-wavelength elements, where one should operate near the resonance of the HIS structure. However, for the configurations considered in this work, particularly, the PEC cylinders covered by patch array and Jerusalem cross cloaks, the analytical model is valid even when the grid elements are placed on a thin dielectric cylinder covering the PEC cylinder. This is due to the fact that, in this scenario we operate at very low frequencies, far from the HIS resonance, making the cloak design very robust. Hence, even for a small thickness of the dielectric spacing between the cloak and the conducting cylinder, very good agreement is obtained between analytical and numerical results.

FIG. 15. Similar to Fig. 12, but for a slotted Jerusalem-cross cloak, with $d = 0.1\lambda_0$, and $b = 0.05\lambda_0$, $b = 0.2\lambda_0$, and $b = 0.5\lambda_0$.

Finally, we study moderately larger objects, in order to analyze the dependence of the previous conclusions on the size of the object. We consider two dielectric cylinders with radii $a = \lambda_0/6$ and $a = \lambda_0/4$. We first designed inductive mantle cloaks with the required values of $X_s$ based on the inductive mesh-grid with geometry shown in Fig. 3. The parameters
FIG. 16. Electric field distribution $E_z$ around a patch array cloak covered conducting cylinder illuminated by a line source located at $d = 0.1\lambda_0$, as a function of azimuthal angle $\phi$ at design frequency $f_0$ for different observation positions: $b = 0.05\lambda_0$, $b = 0.2\lambda_0$, and $b = 0.5\lambda_0$. The analytical results are represented by solid lines and full-wave simulation results by dashed lines.

FIG. 17. Similar to Fig. 16, but for a Jerusalem cross cloak, with $d = 0.15\lambda_0$, and $b = 0.05\lambda_0$, $b = 0.2\lambda_0$, and $b = 0.7\lambda_0$. of the mesh-grid cloaks, along with the considered position of the line source (consistent with the considerations studied above), are given in Table II, obtained using the analytical
FIG. 18. Analytical results of magnitude of E-field distributions on the $\phi$-plane, for a dielectric cylinder with $\varepsilon_r = 4.4$, $d = 0.1\lambda_0$, and radius $a = \lambda_0/6$: (a) with the mesh-grid cloak and (b) without the cloak; for a dielectric cylinder with $\varepsilon_r = 3.9$, $d = 0.1\lambda_0$, and radius $a = \lambda_0/4$: (c) with the mesh-grid cloak and (d) without the cloak. The red colored dots indicate the position of the line source, the white circles in (b) and (d) show the boundaries of the dielectric cylinder without the cloak, and the red dashed circles in (a) and (c) show the presence of the mesh-grid cloak around the dielectric cylinder.

grid-impedance expression given in Refs. 17–19. In Fig. 18 we show the analytical results for the two cases with and without covering the mesh-grid cloak. Comparing the cloaked and uncloaked cases, the first obvious conclusion is that the mantle cloak is still performing well, restoring the cylindrical wave fronts, despite the larger electrical size of the objects. We have confirmed that also in this case the results are in excellent agreement with full-wave simulations.
TABLE II. Parameters of the mesh-grid cloak conformal to large dielectric cylinders.

<table>
<thead>
<tr>
<th>a</th>
<th>N</th>
<th>D</th>
<th>w</th>
<th>(\varepsilon_r)</th>
<th>(a_c)</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda_0/6)</td>
<td>10</td>
<td>(\lambda_0/10)</td>
<td>(\lambda_0/83)</td>
<td>3.9</td>
<td>(a)</td>
<td>0.1(\lambda_0)</td>
</tr>
<tr>
<td>(\lambda_0/4)</td>
<td>15</td>
<td>(\lambda_0/10)</td>
<td>(\lambda_0/154)</td>
<td>4.4</td>
<td>(a)</td>
<td>0.1(\lambda_0)</td>
</tr>
</tbody>
</table>

To conclude this section, it is worth mentioning that, in order to improve the accuracy of the analytical model in the case in which the line source is placed in very close proximity of the cylinder, we can increase the number of unit-cells \(N\), correspondingly decreasing the period \(D\) of the sub-wavelength elements. In our full-wave simulations, in which the granularity of the surface is fully considered, the azimuthal position of the line source may have an effect on the cloaking performance. We have studied this effect, without finding any relevant difference, due to the sub-wavelength size of the inclusions in all the cases considered.

V. LOW-PROFILE CONFORMAL ANTENNA APPLICATIONS

As an example to show the dual nature of the metasurface cloak, in this section, we briefly analyze the electromagnetic radiation of an electric line source placed near an infinite metallic cylinder coated with a conformal patterned metasurface. The idea behind this concept is based on the relevant interest in boosting the radiation from confined sources using metamaterials, as proposed in Ref. 27 and 28. Since the metasurface cloaks effectively provide a surface impedance with a wide range of values, it may be possible to achieve sub-wavelength resonances and enhanced radiation at frequencies different from cloaking. Indeed, in Ref. 17 we pointed out that, in addition to cloaking of metallic cylinders, there were frequency windows in which patterned metasurface cloaks may provide enhanced scattering for plane-wave excitation. Due to the frequency dispersion of the considered surfaces, the coated cylinders indeed showed resonant enhancement of the scattering width (SW), compared to the uncoated cylinders at frequencies different from the cloaking condition\(^ {17}\). By applying the analytical model developed above, here we study the far-field radiation properties of the line source in presence of a conformal patterned surface covering a metallic cylinder, which may
be of great interest to enhance and tailor the radiation from confined sources. The cover is realized using sub-wavelength patch arrays, whose parameters can be found in Table I, and the line source is located at $d = 0.1\lambda_0$. We found in Ref. 17 [see Fig. 17(a)] that the same cloak design analyzed above indeed provides a resonant enhancement of the SW at a larger frequency $f/f_0 = 1.33$. Here we analyze if this same design may be used to improve the far-field radiation properties of a line source placed near the coated cylinder. Fig. 19 compares the directivity radiation pattern with and without the presence of the surface, indeed showing significant improvement in the directivity and radiation efficiency. It should be noted that, by using the same patch array cover with same parameters, the cloaking effect can still be realized at the design frequency ($f_0$), as shown in Section III B.

VI. CONCLUSIONS

We have investigated here mantle cloaks applied to dielectric and conducting cylinders covered by various realistic printed and slotted conformal sub-wavelength periodic elements, illuminated by an electric line source placed close to the object to be cloaked. It has been shown that the analytical design rules derived for plane-wave incidence are still valid for near-field line source excitation. A detailed study has been performed to understand the validity
and limitations of the analytical model for cylindrical conformal mantle cloaks placed in the near field of a line source. The analytical results are in good agreement with full-wave results for all the considered cases, except for the extreme cases in which source and/or observation points are located very close to the periodic surfaces. In addition, mantle cloaking has been proven effective for moderately larger cylindrical objects illuminated by an electric line source. Applications to low-profile conformal antennas have also been demonstrated and discussed. The analysis presented in this work can be extended to study cloaking in the presence of a magnetic line source by using duality.

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REFERENCES

1. A. Alù and N. Engheta, Phys. Rev. E, 72, 016623 (2005).
2. A. Alù and N. Engheta, Phys. Rev. Lett., 102, 233901 (2009).


P. Y. Chen and A. Alù, ACS NANO 5, 5855 (2011).


