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Analysis of Transient Beam Behavior with Impact

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EXTENDED ABSTRACT

After the 9/11 terrorist attacks, more researchers have been focused on the dynamic response of structures under unexpected loads. Extreme loads such as explosions and earthquakes can cause enormous human and infrastructure losses. On the other hand, machinery can routinely experience internal contact. Due to the increased energy needs of today, the wear of nuclear generator tubes and reactors control rods has great significance. Unpredicted failure may be caused vibration-induced impact and is highly dangerous for both equipment and safety. For these reasons, the research of these beam-like structures under extreme loads is of importance.

In this study, the dynamic response of an Euler-Bernoulli beam with adjustable boundary and contact conditions is investigated. Illustrated in Fig. 1, the beam has a rectangular cross-section with depth *b*, height *h*, and second moment of area *I*. Other parameters of the beam are the flexural rigidity *EI*, mass density ρ, and longitudinal length *L*. Torsional springs K_{t1} and K_{t2} and translational springs k_1 and k_2 form the adjustable boundaries. The parameter *k* represents the effective stiffness of the contact interface, and the parameter *a* identifies the location of the contactor from the left hand side of the beam. The deflection of the beam is denoted as *w(x,t)*.

In the numerical model, the transient forcing function is temporally discretized as in Fig. 2: the shock pressure is simplified into constant distributed longitudinal loads. The initial conditions for each interval are the terminal conditions for the previous interval. Impact is induced by the contact element *k* at varying *a*. Using small discrete time steps, the deflection at the contact location *w(a,t)* is monitored to determine whether the system is in contact or not.

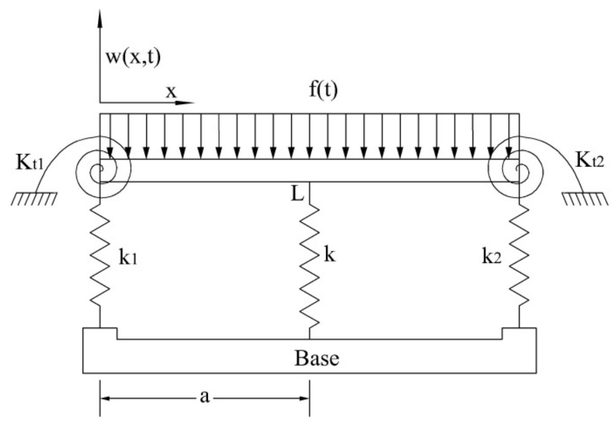

Figure 1. Beam model with adjustable boundary conditions and variable contact.

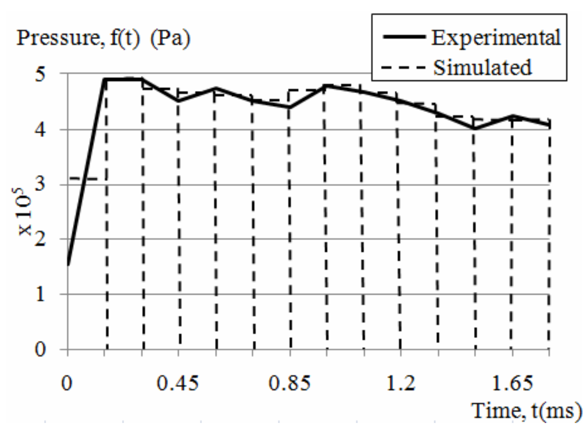

Figure 2. Example of force discretization with an experimental shock loading.

In the analysis algorithm coded in MATLAB, motion of the beam structure is divided into two states T^f and T^c within each vibration is linear. State T^f refers to the "free" condition that the structure is not in contact with the contactor; T^c refers to the in-contact state. Modal analysis is applied to both states, and then compatibility conditions on displacement and velocity at the contact point are applied. To map between the two states, a transform matrix is employed; this relation is derived from the compatibility at the impact or rebound time point and is the projection of each state's modes onto the other state. This process is derived in [1].

At first, an instantaneous impulse force is considered. In order to achieve the most accurate and efficient result, a convergence study is performed to reach the optimal number of included modes. In this study, a reference system will be used as the basis for later parameter studies. The reference beam has a length *L* of 152.4 mm, depth *b* of 10.16 mm, height *h* of 3.05 mm, material density *ρ* of 2700 kg/m^3, and elastic modulus *E* of 0.689 Gpa. The contactor is centered at *a* = 76.2 mm and has a stiffness *k* of 875.6 N/m. No damping is considered for this case.

Modal convergence was examined for three boundary conditions of this reference case. As shown in Fig. 3, eight free and sixteen contact modes are most accurate for a cantilever. This result is similar for both the simply-supported and fixed-fixed conditions.

Five influential parameters will be further investigated through numerical studies. The parameters of interest include structural stiffness as beam height *h*, material as elastic modulus *E*, contact stiffness *k*, contact location *a*, and damping ratio ζ. Their effects on different boundary conditions will be contrasted as well.

Reference

[1] Elizabeth K. Ervin, J. A. Wickert, Repetitive impact response of a beam structure subjected to harmonic base excitation. *Journal of Sound and Vibration* 307(1-2) (2007), 2-19.

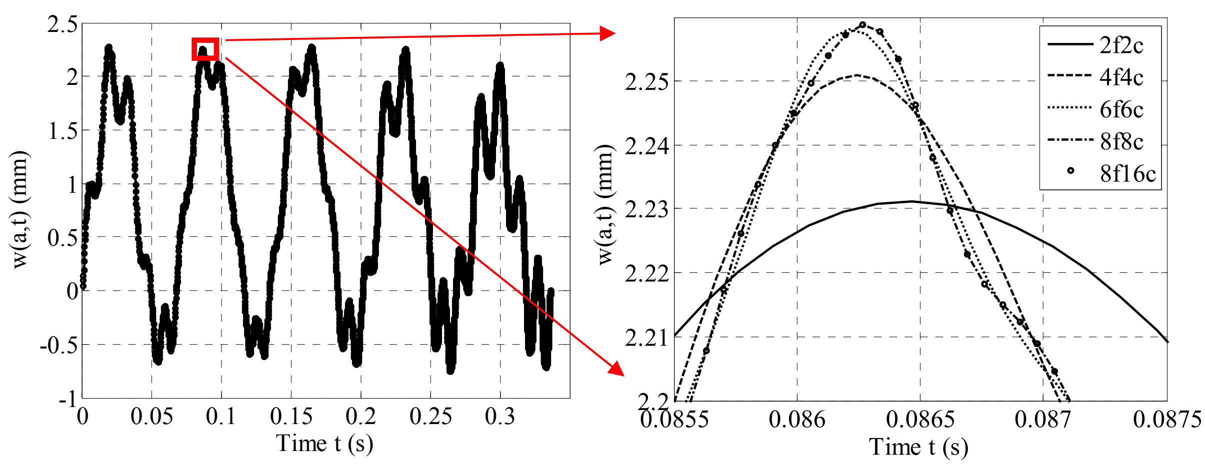

Figure 3. Convergence study for the reference case with fixed-free boundary conditions with an applied impulse of 0.1 milliseconds.