

## Simulated Repetitive Impact in Orthogonal Continuous Structures

Elizabeth K. Ervin

Assistant Professor, The University of Mississippi  
P. O. Box 1848, University, MS 38677, U.S.A.  
eke@olemiss.edu , <http://home.olemiss.edu/~eke/>

**Key Words:** *Impact, Contact, Structural Response, Nonlinear Vibration.*

### ABSTRACT

Structural contact and response interaction occurs in the mechanisms of many fields, including the nuclear, petroleum, biomedical, and automotive industries. Due to manufacturing or by design, a variety of machinery is susceptible to repetitive impact that can generate noise, wear, and damage as well as unexpected frequency-dependent behavior. Structural response and the force input are specifically important in cases of progressive collapse, such as the World Trade Center on September 11, 2001. With this motivation, for instance, Teng and Wierzbicki [1] addressed the successive contact of flexible bodies to determine the critical number of impacts to cause local shear failure. While their focus was tensile strain and critical impact velocity, this work aims to predict the impulsive force through structural response modeling.

This research analytically investigates the repetitive impact dynamics of two orthogonal pinned-pinned beams subjected to base excitation at specified frequency and acceleration. The orthogonal beam configuration restricts the contact to a single point, and the contact interface is modeled by a spring. This ideal contact model is sufficient since the main objective is response prediction rather than wear analysis. While many approaches have been developed for multi-body dynamics, the constraint and modal mapping method is efficiently applied herein to obtain the forced response through modal analysis. The vibration is described in a piecewise linear fashion as sequentially switching between the in-contact and not-in-contact states through an extended operator formulation. The conjoined mode shapes and their orthogonality has been derived in detail to justify this methodology. The temporal gap function is monitored, and compatibility conditions are applied at the times of contact and rebound. The normalized contact impulse  $I^*$  is used to describe the structures' complex interacting behavior through repetitive impact frequency response functions. This simulation methodology has been verified via experimental case study comparisons for a single beam experiencing point contact [2]. Steady state response spectra accurately correspond between the single beam and two beam models.

Simulations illustrate that the associated steady-state repetitive impact behavior of interacting structures is quite complex. Based upon the excitation frequency  $\omega^*$  as normalized by the first not-in-contact natural frequency, the frequency responses include such phenomena as harmonic resonances, bifurcations, grazing impacts, and aperiodicity. In order to determine major response factors, parameter studies are performed on contact stiffness, relative beam stiffness, contact location, modal damping, and stand-off

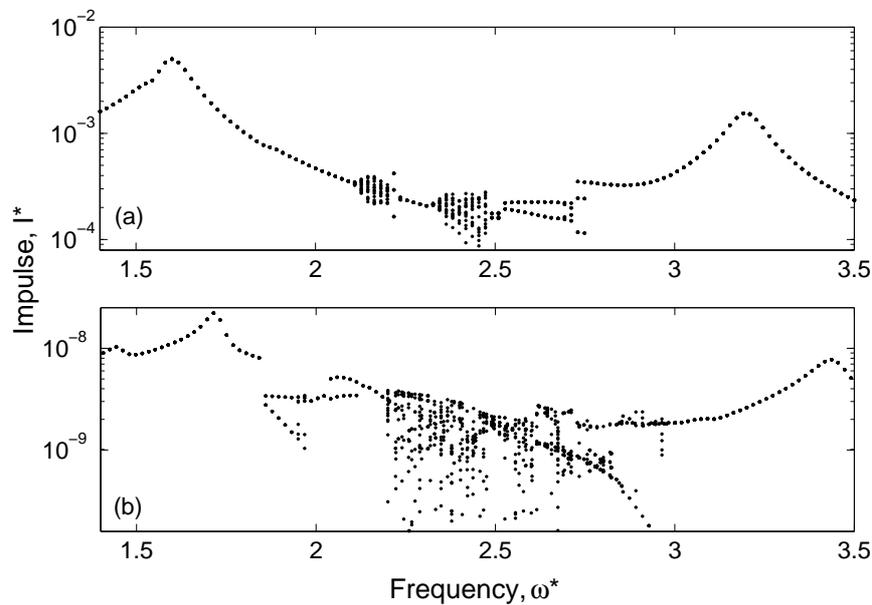


Figure 1: The simulated repetitive impact frequency response functions for a relatively (a) soft and (b) stiff normalized contact stiffness.

gap. The response complexity is strongly affected by contact stiffness and relative beam stiffness but weakly affected by damping, clearance, and acceleration.

The contact stiffness is a significant factor in the complexity of the beam system's response: increasing contact stiffness increases the sophistication of the repetitive impact frequency response functions. For a low contact stiffness, the in-contact mode shapes are lightly coupled. As this contact stiffness increases, the repeated modes involving the beams' symmetric displacements separate into two modes with differing natural frequencies and the beams' mode shapes become more coupled. The repeated modes involving the beams' anti-symmetric modes remain the same if the contact element is located at a nodal point. In the simulations, the contact stiffness  $k^*$ , as normalized by beam static stiffness, is increased from 1 (Fig. 1a) to  $10^5$  (Fig. 1b) such that the first in-contact natural frequency increases by 5.2%. As the contact stiffness increases, double peaks occur near the system resonance, and a greater region of aperiodic motion are evident.

The trends observed in this work will be verified via laboratory experimentation and simulation comparison. The most illuminating result is that proper selection of relative beam stiffness can create passive vibration control. The work will also be expanded to shock and ballistic loading relating to structural survivability. As Lynn and Isobe [3] show, an orthogonal beam-column in a framed structural system exhibits fracture and progressive collapse due to shock wave flow initiated by an impact.

## REFERENCES

- [1] X. Teng and T. Wierzbicki. "Multiple impact of beam-to-beam". *Intl. J. of Impact Engr.*, Vol. **31**, 185–219, 2005.
- [2] E. K. Ervin and J. A. Wickert. "Experiments on a beam-rigid body structure repetitively impacting a rod". *Nonlinear Dynamics*, doi 10.1007/s11071-006-9180-3, 2007.
- [3] K. M. Lynn and D. Isobe. "Structural collapse of framed structures under impact loads using ASI-Gauss finite element method". *Intl. J. of Impact Engr.*, Vol. **34**, 1500–1516, 2007.