FDTD/PBC Algorithm for Skewed Grid Periodic Structures

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Abstract

In this paper, an efficient finite-difference time-domain (FDTD) algorithm with a simple periodic boundary condition (PBC) is presented to analyze periodic structures with arbitrary skewed grid. The algorithm is easy to implement and efficient in both memory usage and computation time. The stability criterion of the algorithm is angle independent and therefore it is suitable for implementing incidence with angle close to grazing as well as normal incidence. The validity of this algorithm is verified through numerical examples with a Jerusalem cross frequency selective surface (FSS) configuration.

Introduction

Periodic structures are of great importance in electromagnetics due to their wide range of applications in FSS, electromagnetic band gap (EBG) structures, phased antenna arrays, periodic absorbers, and negative index metamaterials. FDTD algorithm has been utilized to analyze these structures, and various implementations of PBC have been developed such that only one unit cell needs to be analyzed instead of the entire structure. In [1] a simple and efficient FDTD/PBC algorithm was introduced. In this approach the FDTD simulation is performed by setting a constant horizontal wavenumber instead of a specific angle of incidence. The idea of using constant wavenumber in FDTD was originated from guided wave structure analysis and eigenvalue problems in [2], and it was extended to the plane wave scattering problems in [3]. The approach offers many advantages, such as implementation simplicity, same stability condition and numerical errors as conventional FDTD, computational efficiency near the grazing incident angles, and the wide-band capability. It’s worthwhile to point out that most previous PBCs are developed to analyze axial grid periodic structures. However, there are numerous applications where the grid of the periodic structures has a general skewed angle. Figure 1 shows the geometries of both axial and skewed grid structures. The axial periodic structures are special case of the general skewed grid structures, where the skew angle \( \alpha = 90^\circ \). In this paper, the constant horizontal wavenumber approach is extended to analyze the periodic structures with skewed grid. The new algorithm is very efficient and simple, and it retains the broadband capabilities of the FDTD. The developed FDTD updating equations are summarized first, then several numerical examples including a Jerusalem cross FSS are provided. The numerical results show good agreement with other numerical results obtained from a frequency domain method using Ansoft Designer [4].
The constant horizontal wavenumber approach is extended to analyze the skewed grid periodic structure. According to the ratio of the skewed shift over the FDTD cell size, the general skewed structure in Fig. 1b is divided into two groups: the coincident skewed shift and the non-coincident skewed shift. Figure 2a shows the FDTD grid for the coincident skewed shift periodic structure, while Fig. 2b shows the FDTD grid for the non-coincident skewed shift periodic structure. In this specific example the unit cell of the periodic structure is discretized using 5×5 FDTD grid cells in x and y directions; the unit A is the one to be simulated, while unit B and unit C are the neighboring periodic units. The structure has periodicity of $P_x$ in the x-direction and $P_y$ in the y-direction. $S_x$ is the skewed shift which can be calculated as $S_x = P_y / \tan(\alpha)$, where $\alpha$ is the skew angle. It can be noticed from Fig. 2a that in this case $S_x$ is an integer multiple of the discretization step in the x-direction ($\Delta x$) (coincident case). While from Fig. 2b $S_x$ is not an integer multiple of the discretization step in the x-direction ($\Delta x$) (non-coincident case).

![Fig. 1. Geometries of (a) axial and (b) skewed periodic structures.](image)

![Fig. 2. FDTD grid for skewed periodic structure: (a) with skewed shift is coincident with the FDTD grid, (b) with skewed shift is non-coincident with the FDTD grid.](image)

No modification is needed for magnetic field components and for the electric field components located inside the boundary of cell A. The components on the boundaries will be updated using PBC equations based on the new approach. In this example, the skewed shift is in the x-direction so only $E_x$ and $E_z$ components will be affected by the shift. A similar procedure can be used if the skewed shift is in the y-direction. To update the $E_x$ on the boundary $y = 0$, magnetic field components $H_y$ outside unit A are needed. Due to periodicity and taking into account the skewed shift, one can use magnetic field components $H_y$ inside unit A as follows: for $i + (S_x/\Delta x) \leq n_x$

$$H_x^{n+1/2}(i, 0, k) = H_x^{n+1/2}(i + S_x/\Delta x, n_y, k) \times e^{j k_y S_x} \times e^{j k_y P_y},$$

for $i + (S_x/\Delta x) > n_x$.
where \( n_x \) and \( n_y \) are the total number of cells in \( x \) and \( y \)-directions, respectively. The two exponential terms are used to compensate the phase delay due to the oblique incidence. Using (1) and (2) we can write the updating equation for the \( E_x \) components on the boundary \( y = 0 \) [5]:

\[
E_x^{i+1}(i,1,k) = E_x^i(i,1,k) + \frac{\Delta t}{\Delta y} \times [H_x^{i+1/2}(i,1,k) - H_x^{i-1/2}(i,0,k)] + \frac{\Delta t}{\Delta x} \times [H_y^{i+1/2}(i,1,k) - H_y^{i+1/2}(i,1,k-1)].
\] (3)

The updating equation for \( E_x \) components on the boundary \( y = P_y \) can be written as:

for \( i - (S_x / \Delta x) \leq 0 \)

\[
E_x^{i+1}(i,n_y + 1,k) = E_x^{i+1}(i+n_y - \frac{S_x}{\Delta x},1,k) \times e^{-jk_y(P_y - n_y)} \times e^{-jk_yP_y},
\] (4)

while for \( i - (S_x / \Delta x) > 0 \)

\[
E_x^{i+1}(i,n_y + 1,k) = E_x^{i+1}(i - \frac{S_x}{\Delta x},1,k) \times e^{-jk_yS_y} \times e^{-jk_yP_y}.
\] (5)

The \( E_z \) component can be updated similarly.

As for the non-coincident case, to update the \( E_x \) component in cell 1 (shown in the left top corner in Fig. 2b), an interpolation between \( H_z \) in cell 2 and in cell 3 is needed to get the corresponding \( H_z \) for this \( E_x \) component. A linear interpolation is used according to the two distances \( x_1 \) and \( x_2 \).

\[
H_z^{i+1/2}(1,0,k) = [w_1 H_z^{i+1/2}(1 + \left\lfloor \frac{S_x}{\Delta x} \right\rfloor, n_y, k) + w_2 H_z^{i+1/2}(\left\lfloor \frac{S_x}{\Delta x} \right\rfloor, n_y, k)] \times e^{jk_yS_y} \times e^{jk_yP_y},
\] (6)

where \( \left\lfloor \cdot \right\rfloor \) is the ceiling function, \( w_1 \) and \( w_2 \) are two weighting factors calculated based on distance \( x_1 \) and \( x_2 \) as: \( w_1 = x_1 / \Delta x, w_2 = x_2 / \Delta x \). Using (6) and (3) the \( E_x(1,1,k) \) can be updated. Similarly all other \( E_x \) and \( E_z \) for the non-coincident case can be updated.

**Numerical Results**

In this section, numerical results generated using the new algorithms are presented. The FDTD code was developed in MATLAB programming language and run on a computer with an Intel Core 2, 2.66 GHz processor. The algorithm is used to analyze an FSS structure consisting of Jerusalem cross elements. The periodicity is 15.2 mm in both \( x \) and \( y \) directions [6] and a skew angle of 80 degrees (general non-coincident case). The dimensions of the elements are labeled in Fig. 3. The structure is illuminated by a TE\(^z\) plane wave. Figure 4a provides results for normal incidence (polarization along \( y \)-axis) and Fig. 4b provides results for oblique incidence (\( \theta = 60^\circ \) and \( \phi = 45^\circ \)). The results were compared with results obtained from Ansoft Designer. Good agreement between the results generated using Ansoft Designer and results generated using the new algorithm can be noticed for both normal and oblique incidences. The computational time per simulation for skewed code is 4.53 minutes and the memory usage is 0.2 MB, while for Ansoft Designer computational time per simulation is 45 minutes for 30 frequency points and the memory usage is 21 MB using the same computer.

**Conclusion**

This paper presents a new FDTD/PBC approach to analyze the scattering properties of general skewed grid periodic structures. The approach is developed based on constant horizontal wavenumber technique. It is simple to implement and efficient in terms of both computational time and memory usage. The stability criterion is angle independent, and therefore, it is efficient for incidence with angles close to grazing as well as normal
The presented algorithm is capable of calculating the co-polarization and cross-polarization reflection coefficients for different skewed grid periodic structures excited with normal or oblique incidence. The numerical results show very good agreement with results from frequency domain method solutions.

Fig. 3 JC FSS geometry with skew angle $\alpha = 80^\circ$ (all dimensions are in mm).

Fig. 4 Reflection coefficient co-polarization and cross polarization for JC FSS with skew angle $\alpha = 80^\circ$: (a) Normal incident TE$_z$ plane wave ($\theta = 0^\circ$, $\phi = 0^\circ$). (b) Oblique incident TE$_z$ plane wave ($\theta = 60^\circ$, $\phi = 45^\circ$).

References


