Radar Cross-Section of Targets Loaded with Metamaterial

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Abstract- A rigorous semi-analytical solution is presented for electromagnetic scattering from an array of parallel-coated circular cylinders of arbitrary radii and positions due to an obliquely incident plane wave excitation. Circular metamaterial cylinder/cylinders with metamaterial coating are then used to show the effect of metamaterial characteristics in altering the forward and backward scattering cross-section of arbitrary shaped two dimensional targets. Furthermore, characteristics of metamaterial are used to enhance or reduce the scattered field in pre-specified directions.

1. INTRODUCTION
The analyses of an obliquely incident plane wave scattering from an array of parallel-coated circular cross-section cylinders can be used to study the radar cross-section of two-dimensional scattering object that can be constructed from an array of parallel circular cylinders\cite{1-3}. The scattering of an obliquely incident plane wave on an array of parallel-coated circular cylinders is considered for TM\textsubscript{z} polarization in \cite{4}. The core cylinders and the coating layers can be any of three different materials; metamaterial, dielectric, perfect electric conductor or a combination of two of them. A brief summary of this technique is introduced first, followed by a proof of the validity of the results by comparing the obtained scattered fields with that based on results reported in \cite{5,6} for the special case of normal incidence.

This developed semi-analytical solution for an array of either metamaterial or conductor coated with metamaterial cylinders is used in this paper to show the effects of metamaterial in focusing the field in either the forward or the backward directions, or in any pre-specified direction based on the desired application. Furthermore, to enhance the focusing of received signals by two dimensional reflector antennas, a small number of metamaterial cylinders are placed in strategic positions adjacent to the reflector surface to increase the strength of the field at the feed/receiver element. As an example, significant enhancement of focused field in front of a corner reflector antenna is achieved by loading the reflector surface by two metamaterial cylinders.

2. FORMULATION
The scattering from an obliquely incident E-polarized TM\textsubscript{z} plane wave from an array of M parallel-coated circular cylinders parallel to each other and to the z-axis is considered in a global coordinate system (\(\rho, \phi, z\)). The incident electric field of a plane wave on cylinder “i” is expressed in the (\(\rho, \phi, z\)) cylindrical coordinate system for \(e^{i\omega t}\) time dependence as
\[
E^{\text{inc}}_{z_i}(\rho_i, \phi_i, z) = E_0^e \text{e}^{jk_0 z \cos \theta_0} \text{e}^{jk_0 \rho_i \sin \theta_0 \cos(\phi - \phi_0)} \text{e}^{-jk_0 \rho_i \sin \theta_0 \cos(\phi - \phi_0)} = E_0^e \text{e}^{jk_0 z \cos \theta_0} \text{e}^{jk_0 \rho_i \sin \theta_0 \cos(\phi - \phi_0)} \sum_{n=-\infty}^{\infty} j^n J_n \left( k_0 \rho_i \sin \theta_0 \right) e^{jn(\phi - \phi_0)},
\]

where \( E_0^e = E_0 \sin \theta_0 \), \( \theta_0 \) is the oblique incident angle as shown in Fig. 1, and \( E_0 \) is the amplitude of the incident electric field component. The parameter \( k_0 \) is the free space wave number, \( \phi_0 \) is the angle of incidence of the plane wave in the \( x-y \) plane with respect to the negative \( x \)-axis, and \( J_n(\xi) \) is the Bessel function of order \( n \) and argument \( \xi \). The second expression of the incident field component is in terms of the cylindrical coordinate of the \( i^{th} \) cylinder, whose center is located at \( (\rho_i', \phi_i', z) \) of the global coordinate \( (\rho, \phi, z) \).

The resulting \( z \) component of the scattered electric field from the \( i^{th} \) cylinder can be expressed as

\[
E^s_{z_i}(\rho_i, \phi_i, z) = E_0^e \text{e}^{jk_0 z \cos \theta_0} \sum_{n=-\infty}^{\infty} A_n H^{(2)}_n \left( k_0 \rho_i \sin \theta_0 \right) e^{jn(\phi - \phi_0)}. 
\]

where \( H^{(2)}_n(\xi) \) is the Hankel function of the second type of order \( n \) and argument \( \xi \). The transmitted \( z \) component of the field inside the core cylinders and the coating layers can be expressed, respectively, as

\[
E^{d1}_{z_i}(\rho_i, \phi_i, z) = E_0^e \text{e}^{jk_0 z \cos \theta_0} \sum_{n=-\infty}^{\infty} D_n J_n \left( k_0 \rho_i \sqrt{\frac{k_0^2}{k_0^2} - \cos^2 \theta_0} \right) e^{jn(\phi - \phi_0)},
\]

\[
E^{d2}_{z_i}(\rho_i, \phi_i, z) = E_0^e \text{e}^{jk_0 z \cos \theta_0} \sum_{n=-\infty}^{\infty} B_n J_n \left( k_0 \rho_i \sqrt{\frac{k_0^2}{k_0^2} - \cos^2 \theta_0} \right) + C_n N_n \left( k_0 \rho_i \sqrt{\frac{k_0^2}{k_0^2} - \cos^2 \theta_0} \right) e^{jn(\phi - \phi_0)},
\]

Figure 1. The parameters describing the obliquely incident TMz plane wave on a coated cylinder.
where \( N_n(\xi) \) is the Neumann function of order \( n \) and argument \( \xi \), \( k_{d1} \) is the wave number inside the core cylinders, and \( k_{d2} \) is the wave number inside the coating layers material. The corresponding magnetic field components are given as

\[
H_z^{\prime} (\rho, \phi, z) = E_0 e^{ik_z z} \sum_{n=-\infty}^{\infty} A_n e^{in(\xi - \phi)} \left( k_0 \rho \sin \theta_0 \right) e^{in(\xi - \phi)},
\]

\[
H_{z_1}^{d1} (\rho, \phi, z) = E_0' e^{ik_z z} \sum_{n=-\infty}^{\infty} D_n' e^{in(\xi - \phi)} \left( \frac{k_{d1}^2}{k_0^2} \cos^2 \theta_0 \right) e^{in(\xi - \phi)},
\]

\[
H_{z_1}^{d2} (\rho, \phi, z) = E_0 e^{ik_z z} \sum_{n=-\infty}^{\infty} B_n e^{in(\xi - \phi)} \left( \frac{k_{d2}^2}{k_0^2} \cos^2 \theta_0 \right) e^{in(\xi - \phi)}.
\]

Using Maxwell’s equations, the \( \phi \) components of the electric and magnetic fields can be obtained in all regions.

In this work, we consider only double negative (DNG) metamaterial with negative values of the permittivity and permeability. To show the effect of metamaterial, the wave number and the intrinsic impedance for the \( i \)th layer can be expressed as

\[
k_i = k_0 n_i, \]

\[
n_i = \sqrt{\mu_i \varepsilon_i} \quad \text{for dielectric},
\]

\[
n_i = -\sqrt{\mu_i \varepsilon_i} \quad \text{for metematerial},
\]

\[
\eta = \eta_0 \sqrt{\mu_i / \varepsilon_i}.
\]

The coefficients \( A_{in}, B_{in}, C_{in}, D_{in}, A_{in}', B_{in}', C_{in}' \), and \( D_{in}' \) are unknowns to be determined. These unknowns can be obtained by applying the appropriate boundary conditions on the surface of all core cylinders and the coating layered cylinders.

The boundary conditions on the surface of the \( i \)th core cylinders with \( \rho = a_i, 0 < \theta < 2\pi \) are given by

\[
E_{z_i}^{d1} = E_{z_i}^{d2},
\]

\[
H_{z_i}^{d1} = H_{z_i}^{d2},
\]

\[
E_{\phi}^{d1} = E_{\phi}^{d2},
\]

\[
H_{\phi}^{d1} = H_{\phi}^{d2}.
\]

The boundary conditions on the surface of the \( i \)th coating layers with \( \rho = b_i, 0 < \theta < 2\pi \) are given by

\[\text{...}\]
After some mathematical manipulations and substituting both z and $\phi$ components of the electric and magnetic fields into the boundary condition equations (9-10), the unknown coefficients $A_{\text{in}}$, $B_{\text{in}}$, $C_{\text{in}}$, $D_{\text{in}}$, $A'_{\text{in}}$, $B'_{\text{in}}$, $C'_{\text{in}}$, and $D'_{\text{in}}$ can be determined. A detailed derivation for the unknown field coefficient is reported in [4].

3. Numerical Results

The 2-D echo width of an array of metamaterial cylinders excited by an obliquely incident TM$_z$ plane wave is defined as

$$\sigma_{2D} = 10 \log \left( \lim_{\rho \to \infty} 2\pi \rho \left| \frac{E^f}{E^i} \right|^2 \right).$$

In order to prove the validity of the presented formulation for multiple metamaterial cylinders, the echo width of one, two and three metamaterial cylinders in a linear array configuration located along the x-axis is calculated at 300 MHz. The metamaterial cylinders have relative permittivity $\varepsilon_r=-4$ and relative permeability $\mu_r=-1$. The radius of the cylinders is $a=1\lambda$ and the center-to-center separation is $d=3\lambda$, and excited by a TM$_z$ plane wave with incident angle $\theta_i=90$ and $\phi_i=90$. The results generated using the developed formulation as shown in Fig. 2, show a complete agreement with those given in Fig. 10 of [5].

Figure 3 shows the echo width of an arbitrary positioned array of five cylinders. The radius of the cylinders is $0.1\lambda$, and the center-to-center separation is $0.5\lambda$ and excited by an incident plane wave at $\theta_i=90$ and $\phi_i=180$. The graph shows the echo width for two different cases; dielectric cylinders with $\varepsilon_r=2.2$, and metamaterial cylinders with $\varepsilon_r=-2.2$ and $\mu_r=-1$. For the two cases the results are in complete agreement with the results published in [5].

To show the near field effect of metamaterial in forward and backward scattering, Fig. 4 shows the near field distribution resulting from the incidence of a TM$_z$ polarized plane wave on an array of five cylinders. The cylinders are placed symmetry around the x-axis with the center of all cylinders located on the y-axis. The radius of all cylinders is $0.1\lambda$ and the distance between the centers is $0.5\lambda$ and excited by an incident plane wave at $\theta_i=90$ and $\phi_i=180$. Figure 4-a shows the near field distribution of an array of conducting cylinders while Fig. 4-b shows that of metamaterial cylinders having relative permittivity $\varepsilon_r=-10$ and relative permeability $\mu_r=-1$. A higher isolation in the forward direction was noticed in the case of metamaterial cylinders. Figure 4-c shows the near field distribution of an array of conducting cylinders with the same radius coated with metamaterial layer of thickness $0.05\lambda$. The result shows a greet focusing of the field in the forward direction. This focusing feature is further used to enhance the operation of corner reflector antennas.
Figure 2. The echo width of an array of one, two, and three metamaterial cylinders of radius $a=1\lambda$.

Figure 3. The echo width results of a TM$_z$ plane wave incident on an arbitrary positioned array of five cylinders.
Figure 4. The near field distribution of an array of five cylinders excited by an incident plane wave at $\theta = 90$ and $\phi = 180$. (a) Conducting cylinders, (b) metamaterial cylinders, and (c) conducting cylinders coated with metamaterial.

Figure 5 shows a 90-degree corner reflector of arm length $1.2\lambda$ simulated by 11 conducting cylinders of radius $0.1\lambda$. To show the effect of metamaterial loading in capturing the incident electric field by the antenna receiving element, two metamaterial cylinders of $\varepsilon_r = -5$ and $\mu_r = -1$ are added in front of the reflector at the middle of each arm. Figures 6-9 show the enhancement in the magnitude of the $E_z$ component in front of the reflector due to the presence of the metamaterial cylinders in comparison with dielectric cylinders of $\varepsilon_r = 5$. Figure 6 represents the case of normal incidence with $\theta = 90$ and $\phi = 0$, while Fig. 7 represents the case of normal incidence with $\theta = 90$ and tilted angle in $x-y$ plane $\phi = 30$. Figure 8 shows the case of oblique incidence with $\theta = 45$ and $\phi = 0$ and Fig. 9 shows the case of oblique incidence with $\theta = 45$ and tilted angle in $xy$ plane $\phi = 30$. Table 1 summarizes the amplitude of the $E_z$ at the receiving element point in the $x-y$ plane (focal point). Figure 10-a shows the echo width of a 90° corner reflector antenna due to a normally incident TMz plane wave. An enhancement of about 5 dB in the backward direction is shown when the corner reflector is loaded with metamaterial or dielectric cylinders. Figure 10-b shows the echo width results for the oblique incidence case. The corner reflector loaded with metamaterial cylinders shows more enhancement in the reflected wave than the unloaded corner reflector and the one loaded with dielectric cylinders.

Figure 5. The configuration of a 90° corner reflector antenna
Figure 6. The $E_z$ near field distribution of a corner reflector excited by an incident plane wave at $\theta=90$ and $\phi=0$.
(a) Conducting reflector, (b) Conducting reflector loaded with two dielectric cylinders (c) Conducting reflector loaded with two metamaterial cylinders.
Figure 7. The $E_z$ near field distribution of a corner reflector excited by an incident plane wave at $\theta_i=90$ and $\phi_i=30$. (a) Conducting reflector, (b) Conducting reflector loaded with two dielectric cylinders (c) Conducting reflector loaded with two metamaterial cylinders.

Figure 8. The $E_z$ near field distribution of a corner reflector excited by an incident plane wave at $\theta_i=45$ and $\phi_i=0$. (a) Conducting reflector, (b) Conducting reflector loaded with two dielectric cylinders (c) Conducting reflector loaded with two metamaterial cylinders.
Figure 9. The $E_z$ near field distribution of a corner reflector excited by an incident plane wave at $\theta_{\text{inc}}=45$ and $\phi_{\text{inc}}=30$.
(a) Conducting reflector, (b) Conducting reflector loaded with two dielectric cylinders (c) Conducting reflector loaded with two metamaterial cylinders.

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Table 1. The amplitude of $E_z$ at the $x$-$y$ plane focal point of a corner reflector antenna.
Figure 10. The echo width results of a TMz plane wave incident on a 90° corner reflector antenna loaded with
dielectric or metamaterial cylinders. (a) $\theta_l=90$ and $\phi_l=0$, (b) $\theta_l=45$ and $\phi_l=0$.

4. Conclusion
The analyses of an obliquely incident plane wave scattering from an array of parallel-coated circular
cross-section cylinders is summarized for TMz polarizations. This solution is verified for metamaterial cylinders,
and can be used to study electromagnetic interaction with two-dimensional scattering object that can be
constructed from an array of parallel circular cylinders. The effect of metamaterial in enhancing the forward or
backward scattering from an array of cylinders was studied, and the effect of isolating or focusing the incident
plane wave field is also demonstrated. Significant enhancement of focused field in front of corner reflector
antennas is achieved by loading the reflector by metamaterial cylinders.

REFERENCES
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