An Improved PSO Method using Gaussian Distributed Random Variables for Electromagnetic Optimization

Lijun Zhang, Fan Yang, and Atef Z. Elsherbeni

Center of Applied Electromagnetic Systems Research (CAESR)
Department of Electrical Engineering
The University of Mississippi, University, MS 38677
lzhang5@olemiss.edu, fyang@olemiss.edu, atef@olemiss.edu

Introduction

Particle Swarm Optimization (PSO) is an algorithm that finds a solution of an optimization problem in a searching space based on swarm intelligence. The particle swarm optimization algorithm was first described in 1995 by Kennedy and Eberhart [1], and in recent years, this technique was successfully employed in many electromagnetic optimization problems [2]. As an application of swarm intelligence, PSO algorithm shares some similarities with the social-psychological principle. When particles search for the “best answer” in a multidimensional space, the social influence and social learning enable each particle to maintain cognitive consistency. A communication scheme is developed in order to connect each particle in the swarm. Within this network, they can exchange information, which is known as “personal best location”, to interact with each other, and to affect the next action of the individual particle.

During the PSO implementation, some random factors are added into the swarm in order to simulate the unpredictable effect in nature swarm behaviors [2-3]. In most current PSO methods, the random factors are generated by a uniformly distributed random function. However, according to statistics theories, many psychological measurements and physical phenomena can be better approximated by a Gaussian distributed random function because of the central limit theorem [4].

In this paper, the Gaussian distributed random variables are introduced into the PSO algorithm to enhance its performance. A comparison between the classic PSO and the improved PSO is presented in order to explore a more effective strategy for the optimization of EM problems.

Gaussian Distributed Random Variables in PSO

Assume we have a population of M particles searching in a N-dimensional space. The most important formula in PSO algorithm is the velocity updating equation:

\[ v_n^i(t + \Delta t) = \omega v_n^i(t) + c_1 r_1(p_n^i - x_n^i(t)) + c_2 r_2(g_n - x_n^i(t)) \]  

(1)
where \( v_n^i \) and \( x_n^i \) are the velocity and position, respectively, of the \( i \)th particle in the \( n \)th dimension; \( \Delta t \) is the time step; \( \omega \) is an inertia factor; \( c_1 \) and \( c_2 \) are factors that determine the relative “pull” of personal best (\( p_n^i \)) and global best (\( g_n \)). The parameters \( r_1 \) and \( r_2 \) are two statistically independent random variables, which are usually generated by a random function uniformly distributed between 0 and 1.

Many statistic studies have indicated that most psychological variables approximately follow a normal distribution. As mentioned before, PSO can also be described by social-psychological principles. Therefore, the Gaussian distributed random variables \( G(\mu, \sigma^2) \) are introduced instead of the uniformly distributed random variables:

\[
v_n^i(t + \Delta t) = \omega v_n^i(t) + c_1 G(\mu, \sigma^2)(p_n^i - x_n^i(t)) + c_2 G(\mu, \sigma^2)(g_n - x_n^i(t)) \tag{2}
\]

Each Gaussian function may be defined by two parameters: the mean ("average", \( \mu \)) and variance ("variability", \( \sigma^2 \)), respectively [6]. In the traditional velocity update equation (1), \( r_1 \) and \( r_2 \) are uniformly distributed between 0 and 1, which have a mean value of 0.5. Thus the same mean value is selected for the Gaussian distributed function \( G(\mu, \sigma^2) \).

Figure 1 shows Gaussian probability density functions (PDF) with three different \( \sigma \) values. When the \( \sigma \) value is increased, the probability between 0~1 will be decreased. Table 1 gives the probabilities of the “point” located in the area of 0~1 with different \( \sigma \). It is obvious that the random variable also has the opportunity to be located in other regions outside of 0~1 but the probability is much smaller as compared with locations in the region 0~1.

![Fig. 1. Probability density function for different sigma.](image)

<table>
<thead>
<tr>
<th>( \sigma ) (sigma)</th>
<th>Probability between 0~1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/3</td>
<td>86.64%</td>
</tr>
<tr>
<td>1/4</td>
<td>95.45%</td>
</tr>
<tr>
<td>1/6</td>
<td>99.73%</td>
</tr>
</tbody>
</table>

Table 1. Probability between 0~1.

**Numerical Results**

In order to demonstrate the validity of the Gaussian distributed variables introduced in the PSO algorithm, two optimization problems are selected: one is a mathematical function, Rastigrin function; and the other one is a typical EM
problem, which is a 10-elements linear antenna array [5]. The Rastigrin function given as: 
\[ f(x) = \sum_{i=1}^{N} (x_i^2 - 10 \cos(2\pi x_i) + 10) \], was used as a testbed with dimension \( N=3 \). It has a minimum value of zero. 16 different \( \sigma \) values are tested from \( \sigma = \frac{1}{6} \) to \( \sigma = \frac{1}{3} \), and their results are compared with that of the uniformly distributed random variable. For each \( \sigma \) value, 100 independent trials are tested to get a statistically reliable result and each trial runs for 100 iterations. The best fitness value of each trial is summed together to obtain an average best fitness of the 100 trials for that specific \( \sigma \) value, as shown in Figure 2. It is obvious to see that the \( \sigma \) value whose fitness is smaller than uniform distribution fitness is considered to be a good choice, otherwise the \( \sigma \) is not a suitable candidate. And the best \( \sigma \) is 0.24 where the smallest fitness value is located. Figure 3 compares the convergence curves between uniform distribution and Gaussian distribution with \( \sigma \) equals to 1/3.8, 1/4 and 1/4.1 separately. It shows how these two kinds of distributions perform in the algorithm. The result shows that Gaussian distribution case outperforms uniform distribution case.

The next experiment is using the Gaussian distribution PSO to optimize the element location of a ten elements unequally spaced linear array with the total length of 5 \( \lambda \). The goal is the suppression of the sidelobe level (SLL) to -18.96dB in the area 0 ~ 78.5 degree and 101.5 ~ 180 degree. Here we chose \( \sigma = 0.24 \), and the optimized antenna pattern is shown in Fig. 4. Also we calculate the average fitness value and its standard deviation (SD) though 100 independent trials. The results are listed in Table 2. Both the average and SD values using Gaussian variables are smaller than that using uniform variable, which means that the Gaussian distribution PSO works better than the uniform distribution PSO.
Table 2. Optimization results of 10 elements array design.

<table>
<thead>
<tr>
<th>10 elements array</th>
<th>Gaussian</th>
<th>Uniform</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Fitness</td>
<td>0.0214</td>
<td>0.0726</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.0577</td>
<td>0.3310</td>
</tr>
</tbody>
</table>

Fig. 4. 10-elements array optimization using the improved PSO method with Gaussian distributed random variables.

Conclusion

Gaussian distributed variables are introduced in the PSO algorithm. Comparison results indicate that the optimization goals for both mathematic and electromagnetic problems are successfully achieved and the new variables in this optimization method work better than traditional uniformly distributed variables.

References