OPTIMIZATION USING TAGUCHI METHOD FOR ELECTROMAGNETIC APPLICATIONS

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ABSTRACT

This paper presents a novel electromagnetic optimization technique based on Taguchi method. Using the concept of the orthogonal array (OA), Taguchi method effectively reduces the number of tests required in an optimization process. Although this method has been successfully applied in many fields such as chemical engineering, mechanical engineering, IC manufacture, power electronics etc., it is not well known to the electromagnetics community, and only limited applications are available. The goal of this study is to introduce Taguchi method to the electromagnetics community and demonstrate its great potential in electromagnetic optimizations. The implementation procedure of the Taguchi method is presented in Fig. 1. The proposed optimization procedure has been applied in designing a linear antenna array with a sector beam pattern and a microstrip band stop filter (BSF). The desired antenna pattern and frequency response of BSF are successfully achieved. Compared to other optimization techniques, such as the genetic algorithm (GA) and particle swarm optimization (PSO), Taguchi method is easy to implement, and can quickly converge to the optimum solution.

1. INTRODUCTION

Optimizations of linear antenna arrays have received great attention in the electromagnetics community for many civilian and military applications. One of the typical examples is the design of a sector beam pattern for wider coverage. They are usually accomplished using the Fourier transform or the Woodward-Lawson methods. This study uses a new electromagnetic optimization technique, Taguchi method [1], to design a linear antenna array that produces a sector beam radiation pattern.

Microwave filters are widely used in telecommunication equipments. A bandstop filter suppresses the noises coming from environment or prevents spurious signals interfering with other systems for a specific frequency band. Although filters designed using lumped elements can realize the desired frequency response, it is difficult to control the lumped elements’ properties within microwave band. Therefore passive printed types of filters are usually used for microwave applications. These filters are planar structure, and are composed of several stubs of microstrip lines. An optimization technique and a full wave EM simulator are necessary tools for an optimum design of such filters.

In this study, a full wave simulator, IE3D [2] is used to analyze a microstrip band stop filter, and Taguchi method is applied as an external optimizer. Efficiency of this design procedure illustrates the excellent optimization performance of the Taguchi method. Detailed procedure will be discussed in this paper.

2. ORTHOGONAL ARRAY

The notation \( OA(N, m, s, t) \) is used to represent an orthogonal array, where \( N \) is the number of experiments, \( m \) is the number of parameters to be optimized, \( s \) is the number of levels, and \( t \) is the OA strength. The array size is \( N \) rows by \( k \) columns, with entries from 1 to \( s \). One of the properties of OA is that all possible combinations of up to \( t \) parameters occur together equally, which ensures a balanced, fair comparison of levels of any parameter or interaction of parameters. In general, one could increase the strength \( t \) of the orthogonal array to consider the interactions of more parameters. However, the larger the strength \( t \) is, the more rows the orthogonal array has. The orthogonal arrays used in this paper have a strength of 2, which was found to be sufficient for the problems considered.

Another useful property of orthogonal arrays is that any \( N \times m' \), sub-array of an \( OA(N, m, s, t) \) is an \( OA(N, m', s, t') \), where \( t' = \min\{m', t\} \). In other words, if one or more columns are deleted from an OA, the resulting array is still an OA but with a smaller number of columns.

This property is especially useful when selecting an orthogonal array with an arbitrary number of parameters.
from an existing OA database. More orthogonal array properties can be found in [3].

Since the orthogonal array is the fractional factorial characteristics, only \( N \) experiments are needed instead of a full factorial strategy, which needs to conduct \( s^n \) experiments. After a simple analysis and process of the output results, an optimum combination of the parameter values can be obtained [1]. It is demonstrated in statistics that although the number of experiments are dramatically reduced, the optimum result obtained from the orthogonal array usage is close to that obtained from the full factorial approach or any other optimization technique as will be shown in this paper.

3. IMPLEMENTATION PROCEDURE OF TAGUCHI METHOD

Taguchi method is performed using an iterative procedure to reach the optimum output or solution, as shown in Fig. 1. Each step in the flow chart is discussed in the following subsections.

3.1 Initializing the Problem

The optimization procedure starts with the problem initialization, which includes the selection of a proper OA, and an appropriate design of the fitness function. The selection of an OA depends on the number of input parameters of the optimization problem, and the number of levels of each parameter. In order to describe the non-linear effect, usually three levels are necessary for each input parameter.

The fitness function is devised according to the optimization goal. The fitness represents how close the current result is to the design goal. In an evaluation algorithm, usually either the maximum or minimum fitness is to be achieved.

3.2 Designing Input Parameters Using an OA

The optimization range of each parameter, and the corresponding numerical values for the three levels of the input parameters should be decided in order to conduct the experiments. For example, for a three levels OA in first iteration, the value for level 2, \( a(n)^2 \), is selected at the center of the computational range, and can be written as:

\[
a(n)^2 = \frac{\text{max} + \text{min}}{2},
\]

where \( \text{max} \) is the upper bound of optimization range, and \( \text{min} \) is the lower bound of optimization range. \( a(n) \) indicates the \( n^{th} \) input parameter, the subscript 1 indicates the 1st iteration, and the superscript 2 indicates the level 2.

The values \( a(n)^1 \) and \( a(n)^3 \) are calculated by adding/subtracting \( a(n)^2 \) with a parameter called level difference \( LD(n) \). The level difference, \( LD(n)_1 \) in the 1st iteration is determined by

\[
LD(n)_1 = \frac{\text{max} - \text{min}}{s + 1}.
\]

Thus

\[
a(n)^1 = a(n)^2 - LD(n)_1, \quad (3)
\]
\[
a(n)^3 = a(n)^2 + LD(n)_1. \quad (4)
\]

Each entry of the OA is then converted into a corresponding value of the input parameter \( a(n)^m \).

3.3 Conducting Experiments

After converting the OA entries to proper input values, the fitness function for each experiment can be calculated analytically or through numerical
simulations. The fitness value is used to calculate the corresponding S/N ratio ($\eta$) in Taguchi method [4] through the following formula:

$$\eta = -20 \log (Fitness) \text{ (dB)}.$$  \hspace{1cm} (5)

Hence, a small fitness value results in a large S/N ratio. After conducting all experiments, the fitness and corresponding S/N ratio are obtained. These results are used to build a response table by averaging the S/N ratios of the same level $m$ for each parameter $a(n)$ and each level $k$ using

$$\eta(m,n) = \frac{s}{N_i \text{, } O(A(n) = m)} \sum \eta_i.$$  \hspace{1cm} (6)

### 3.4 Identifying Optimal Level Values and Conducting Confirmation Experiment

Once the response table is created, the optimal level for each parameter can be identified by finding the largest S/N ratio in the corresponding column of the response table.

Next, a confirmation experiment is performed using the combination of the optimal level values, $a(n)^{opt}$ identified in the response table. This confirmation test is necessary because the OA based experiment is a fractional factorial experiment, and the optimal combination may not be included in the OA. The fitness value obtained from the optimal combination is regarded as the fitness value of the current iteration.

### 3.5 Reducing the Optimization Range

If the results of the current iteration do not meet the termination criteria, which are discussed in the following subsection, the process is repeated in the next iteration. The optimal level values of the current iteration are used as central values (values of level 2) for the next iteration, that is

$$a(n)_{i+1}^2 = a(n)^{opt}_i.$$  \hspace{1cm} (7)

To reduce the optimization range, the $LD(n)$ will be multiplied with a reduced rate ($RR$) to obtain $LD(n)_{i+1}$ for the $(i+1)^{th}$ iteration, thus

$$LD(n)_{i+1} = RR \cdot LD(n)_i.$$  \hspace{1cm} (8)

The $RR$ can be set between 0.5 and 1 depending on the problem. The larger is the $RR$, the slower is the convergence.

If the $LD(n)_{i+1}$ is a large value, and meanwhile $a(n)^2_{i+1}$ is located near the upper bound or the lower bound of the optimization range the $a(n)^1_{i+1}$ or $a(n)^3_{i+1}$ may reside beyond the optimization range. Therefore, a process of checking the level value is necessary in order to guarantee that all level values are located within the optimization range. If the excessive situation happens, reassigning the level value for the parameter will be performed. For example if $a(n)^1_{i+1}$ is smaller than $min$, the $a(n)^1_{i+1}$ is then set to $min$.

### 3.6 Checking the Termination Criteria

When the number of iterations increases, the level difference of each element decreases. Hence, the level values are close to each other and the fitness value of next iteration is close to the fitness value of the current iteration. The following equation can be used as a termination criterion for the optimization procedure:

$$\frac{\text{max}(LD(n)_i)}{LD(n)_i} \leq \text{converged value}.$$  \hspace{1cm} (9)

Usually, the converged value can be set between 0.001 to 0.01 depending on the problem. If the design goals are achieved or Eq. 9 is satisfied, the optimization process will terminate.

### 4. DESIGN OF A LINEAR ANTENNA ARRAY

In this section, Taguchi method is used to design a sector beam pattern to demonstrate the versatility and robustness of the Taguchi method. A 20-element equally spaced array with half wavelength element spacings is used as shown in Fig. 2. Both magnitudes and phases of the array elements are optimized to shape the antenna pattern [4]. Thus, an OA(81, 20, 3, 2) [5], which offers ten columns for magnitudes and ten columns for phases, is adopted in the sector beam pattern synthesis.

![Figure 2. Geometry of a 20-element equally spaced linear array.](image)
The requirements for a sector beam pattern are shown in Fig. 3 using the dashed line. There are two specific regions in the angular range to define the sector beam. Region I ranges from 80º to 100º, and it is required that ripples in this region are smaller than 0.5 dB. Region II controls the side lobe levels, which are all below –25 dB between 0º and 70º, and between 110º and 180º.

For a 20-element symmetrical array, the array factor can be written as:

\[
AF(\theta) = \sum_{n=1}^{10} a(n) \cos[kd(n)\cos(\theta) + \beta(n)],
\]

where \(k\) is the wave number; \(a(n)\), \(d(n)\), and \(\beta(n)\) are the excitation magnitude, location, and phase of the \(n^{th}\) element, respectively.

Taguchi method is used for optimizing the excitation magnitude and phase of each element that have the optimization range of 0 to 1 and -\(\pi\) to \(\pi\), respectively.

The following fitness function can be used in the optimization:

\[
Fitness = \int_{0}^{\pi} |AF_d(\theta) - AF(\theta)| d\theta ,
\]

where the \(AF_d(\theta)\) is the desired sector beam pattern, and \(AF(\theta)\) is the pattern obtained from Eq.10. Basically, the fitness can be seen as the difference area between desired pattern and obtained pattern. The smaller value of the fitness function, the better match between the obtained pattern and the desired one. Equation 11 is used for evaluating the fitness value during the optimization process. The converged value is set to 0.002, and the \(RR\) is set to 0.9.

After 60 iterations, an optimum sector beam pattern is obtained and plotted in Fig. 3. It has a ripple of 0.48 dB in region I, the beam width at –25 dB SLL is 41.2º, and the HPBW is 28.1º. The optimized excitation magnitudes of the elements from number one to number ten are \([0.437, 0.321, 0.188, 0.122, 0.132, 0.130, 0.079, 0, 0, 0]\), as shown in Fig. 4(a). The optimized excitation phases (degree) of the elements are \([9.03, 2.51, -16.74, -77.72, -119.81, -112.63, -111.57, -111.27, -170.14, -175.43]\), as shown in Fig. 4(b). The optimized result indicates that a 14 elements symmetrically linear array is capable to realize the design goal with less number of antenna elements. The convergence curve of fitness value is presented in Fig. 5, which demonstrates the efficiency of the Taguchi method.

![Figure 3](image-url)  
**Figure 3.** Array factor of a 20-element linear array with a sector beam pattern. The dashed line is the desired pattern, and the solid line is the optimized pattern.

![Figure 4](image-url)  
**Figure 4.** Optimized element excitations of the linear antenna array with a sector beam pattern as shown in Fig. 3. (a) The magnitudes of elements and (b) the phases of elements.
Figure 5. Convergence curve of the fitness value of the 20-element equally spaced linear array for the sector beam pattern design.

Figure 6. Geometry of the band stop filter.

5. DESIGN OF A BAND STOP FILTER

A symmetrical double folded stub microstrip band stop filter (BSF) as shown in Fig. 6 is used as an example to demonstrate the validity of Taguchi method in filter design. The thickness of the substrate is 5 mils with relative dielectric constant of 9.9. The characteristic impedance of all microstrip lines is 50 Ω, and the corresponding width \( W \) is 4.8 mils.

The design specifications, which are drawn as the dotted line (\( S_{21,d} \)) in Fig. 7, are taken as

Region I:

\[
\left| S_{21} \right| > -3 \text{ dB for } f < 9.5 \text{ GHz and } f > 16.5 \text{ GHz}
\]

Region II:

\[
\left| S_{21} \right| < -30 \text{ dB for } 12 \text{ GHz} < f < 14 \text{ GHz}
\]

IE3D, which is a full wave electromagnetic solver based on method of moments, is utilized to obtain the \( S \) parameters of the BSF. Taguchi method is applied as the external optimizer to drive the IE3D engine. Each time the dimensions of the BSF is modified based on the values determined by the algorithm of the Taguchi method, the BSF is simulated using the IE3D, and \( S \) parameters are obtained. If the result does not meet the termination criteria, the dimensions are modified for the next iteration process.

In Fig. 6 three parameters \( L_1 \), \( L_2 \), and \( S \) are to be optimized to achieve the design goal of the BSF. An OA(9, 3, 3, 2) [5], which offers three columns for \( L_1 \), \( L_2 \), and \( S \), is adopted in the BSF optimization, and is shown in Tab. 1. Before optimization process, the initial filter dimension of \( L_1 \) is \( L_{10} = 74 \) mils, \( L_2 \) is \( L_{20} = 62 \) mils, and \( S \) is \( S_0 = 13 \) mils.

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<th>( L_1 )</th>
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<tr>
<td>9</td>
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Table 1. A three levels OA(9, 3, 3, 2) with three parameters \( L_1 \), \( L_2 \), and \( S \).

The optimization ranges of these parameters are

\[
L_1 = L_{10} \pm \Delta L_1, \quad (12)
\]

\[
L_2 = L_{20} \pm \Delta L_2, \quad (13)
\]

\[
S = S_0 \pm \Delta S, \quad (14)
\]

where \( \Delta L_1 = 10 \) mils, \( \Delta L_2 = 30 \) mils and \( \Delta S = 9 \) mils.

The following fitness function is used in the optimization:

\[
\text{Fitness} = w_1 \left[ \int_{5}^{9.5} (S_{21,d} - S_{21}) \, df + \int_{16.5}^{20} (S_{21,d} - S_{21}) \, df \right] + w_2 \left[ \int_{12}^{14} (S_{21,d} - S_{21}) \, df \right],
\]

where \( w_1 \) and \( w_2 \) are the weights of Region I fitness and Region II fitness, respectively, the unit of frequency is GHz, and the \( df \) is frequency interval set to 0.15GHz. Basically, the fitness can be seen as the difference area between \( S_{21,d} \) and obtained \( S_{21} \). The smaller the value of the fitness function, the better are the obtain results towards to the desired design goal. The ideal fitness value is zero, which means that the optimized results totally satisfy the design goal. Equation 15 is used for evaluating the fitness value during the optimization process. The value of \( w_1 \) is 5, and \( w_2 \) is 1. The converged value is set to 0.05, and the \( RR \) is set to 0.8.
After 11 iterations, the fitness value reached to zero, and the optimization process ended due to meeting the design goal. The convergence curve of fitness is presented in Fig. 8, which again demonstrates the efficiency of the Taguchi method. The dimensions of initial BSF, obtained by Taguchi method, and the ones optimized by gradient [6] are listed in Table 2.

Table 2. The dimensions of prototype BSF, obtained by Taguchi method, and optimized by gradient.

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<td>L1</td>
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<td>92.26</td>
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<tr>
<td>L2</td>
<td>62.0</td>
<td>84.71</td>
<td>84.64</td>
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<tr>
<td>L3</td>
<td>13.0</td>
<td>4.80</td>
<td>5.12</td>
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The $S_{21}$ of the BSF optimized by Taguchi method, the $S_{21}$ of BSF optimized by gradient [6], the $S_{21}$ of prototype BSF, and the design specification, $S_{21, d}$ are all shown in Fig. 7. The desired frequency responses of $S_{21}$ are achieved, which again demonstrate the validity of the Taguchi method.

6. CONCLUSION

A novel electromagnetic optimization technique based on Taguchi method is introduced in this study. The implementation procedure is described, and a linear antenna array with sector beam pattern and a microstrip band stop filter are discussed to demonstrate the validity of Taguchi method. Optimized results show that the desired sector beam pattern and band stop frequency response are successfully obtained. It is found that Taguchi method is easy to implement and converges to the desired goals quickly. This study shows that the optimization performances of Taguchi method are quite excellent, and can reach the same goals optimized by gradient, GA [7], and PSO optimization techniques.

REFERENCES