FDTD Analysis of Periodic Structures at Arbitrary Incidence Angles: A Simple and Efficient Implementation of the Periodic Boundary Conditions

Fan Yang* (1), Ji Chen(2), Rui Qiang(2), and Atef Elsherbeni(1)

(1) Department of Electrical Engineering, The University of Mississippi, University, MS 38677
(2) Department of Electrical and Computer Engineering, University of Houston, Houston, TX 77204
fyang@olemiss.edu, jchen18@uh.edu, rui.qiang@mail.uh.edu, atef@olemiss.edu

Introduction

Periodic structures are widely used in electromagnetics, such as frequency selective surfaces (FSS), electromagnetic band gap (EBG) structures, and negative index materials. When the finite-difference time-domain (FDTD) method is used to analyze these structures, an important issue is how to implement the periodic boundary conditions (PBC) in the time domain simulation. Various PBCs have been developed in the last decade [1], and recently a spectral FDTD method was introduced in [2] that is capable to analyze arbitrary incidence angles. Starting from the constant wave number concept proposed in [2], this paper describes a simple procedure to implement the periodic boundary conditions in the FDTD simulation. Instead of using auxiliary fields \( P \) and \( Q \), we use directly \( E \) and \( H \) fields. The principle of the approach is discussed and the plane wave excitation is highlighted. The validity of the approach is demonstrated through numerical examples.

FDTD/PBC Algorithm

A. Constant wavenumber method. For a periodic structure with a periodicity of \( a \) along the \( x \) direction, the PBC in the frequency domain is expressed as below:

\[
E(x = 0, y, z) = E(x = a, y, z) \exp(jk_x a),
\]

where \( k_x \) is the horizontal wave number determined by both the frequency and incident angle:

\[
k_x = k_0 \sin \theta = \frac{a \sin \theta}{C}.
\]

When (1) is transformed into the time domain, future time data are needed in the updating equation:

\[
E(x = 0, y, z, t) = E(x = a, y, z, t + a \sin \theta / C), \tag{3}
\]

which is the fundamental challenge in formulating PBC in the FDTD method. To circumvent this problem, a constant wavenumber method was proposed in [2]. If \( k_x \) is a constant, (1) can be transformed into the time domain equation as below:

\[
E(x = 0, y, z, t) = E(x = a, y, z, t) \exp(jk_x a). \tag{4}
\]

Note that \( \exp(jk_\theta a) \) is a constant and no future time data are needed in (4).
B. Plane wave excitation. The implementation of the constant wavenumber method is similar to the normal incidence method [1]. The standard Yee’s scheme is used to update the electric and magnetic fields with time marching. The perfectly matched layers (PML) and the periodic boundary condition in (2) are used to truncate the computational domain in the vertical and horizontal directions, respectively. One important issue of this method is how to excite a plane wave into the computational domain. The traditional method incorporates both tangential E and H fields on the incidence plane. However, this technique is not applicable in the constant wavenumber method. For example, if a TE\textsuperscript{z} polarized plane wave impinges onto a FSS structure as shown in Fig. 1(a), and the xz plane is the incidence plane, the tangential field components are $E_y$ and $H_x$. The magnitude of the latter one depends on the incident angle, which varies with frequency when $k_x$ is fixed. Thus, it is difficult to add $H_x$ in the time domain computation. As a consequence, only $E_y$ component of the incident fields is added on the incidence plane. Similarly, if the incident field is TM\textsuperscript{z} polarized, only $H_y$ component will be incorporated on the incidence plane.

It is also important to point out that additional attention needs to be paid to the time domain excitation waveform. Here, a modulated Gaussian waveform is used with corresponding phase delay along the $x$ direction:

$$E_y^{\text{inc}}(x,t) = \exp\left[-\frac{(t-t_0)^2}{2\tau^2}\right]\exp(j2\pi f_0 t)\exp(-jk_x x).$$

(5)

The parameters of the Gaussian waveform are selected to exclude the frequency components under the light line (see Fig. 1(b)). For a specific value of $k_x$, the minimum allowable frequency for plane wave incidence is determined below:

$$f = \frac{k_0}{2\pi \sqrt{\varepsilon_0 \mu_0}} \sin \theta \geq \frac{k_x}{2\pi \sqrt{\varepsilon_0 \mu_0}} = f_{\text{min}}.$$  

(6)

Thus, the bandwidth of the Gaussian pulse ($\tau_f = 1/\tau_i$) needs to satisfy below equation to avoid spurious fields excited under the light line:

$$3\tau_f < f_0 - f_{\text{min}}.$$  

(7)

C. Advantages of the constant wavenumber method. The proposed approach has two distinct advantages. First, it is easy and straightforward to implement. In contrast to the field transformation method that uses auxiliary fields P and Q, the new approach computes $E$ and $H$ directly. Thus, this method uses the standard Yee’s scheme and PML, hence no need for derivation of complicated formulas.

Second, the new algorithm is efficient to calculate the scattering at large incident angle. When the split field technique is used to calculate the oblique incidence, the time step size needs to be reduced due to the stability constrains. In particular, the simulation time becomes prohibitively long when the incident angle is close to the grazing angle. In contrast, the constant wavenumber method uses a standard Yee’s scheme to update the $E$ and $H$ fields. The stability condition remains
unchanged regardless of the horizontal wave number and incident angle. Thus, there is no need to change the time step size for different incident angles, and the proposed method is efficient for arbitrary incident angles.

Numerical Example

The developed FDTD/PBC algorithm is applied to characterize the reflection coefficient of a frequency selective surface (FSS) consisting of dipole elements. The thin periodic dipole array is mounted on a dielectric slab, as shown in Fig. 1(a). The parameters of the periodic structure are provided in the caption, which is the same as those given in [3]. A TE$^z$ plane wave impinges upon the periodic structure in the x-z plane. The FDTD simulation is performed 100 times using different value of $k_x$ sampled along the axis of the horizontal wavenumber. The computed reflection coefficient is plotted in Fig. 1(b). Strong reflection is represented by the dark red color and transmission region is denoted by the blue zone. The first reflected region occurs around 9 GHz and the frequency of total reflection slightly decreases as $k_x$ increases. The first transmission region starts at 16 GHz for normal incidence ($k_x=0$). When $k_x$ increases, both the transmission frequency and the magnitude of the transmission coefficient decrease. It is also observed from Fig. 1(b) that the second reflection region immediately follows the first transmission region. Transmission and reflection at multiple Floquet modes occur at frequency higher than 16 GHz.

![Fig. 1. (a) Geometry of a dipole FSS on a dielectric slab. The dipole length is 12 mm and width is 3 mm. The periodicity is 15 mm in both x and y directions. The substrate has a thickness of 6 mm and dielectric constant of 2.2. (b) The reflection coefficient ($\Gamma$) of the dipole FSS computed using the constant wavenumber method.](image)

Using the relation between the wavenumber and incident angle in (2), it is easy to extract curves of the reflection coefficient versus frequency at any incident angle. For example, the reflection coefficients for the incident angles of 0°, 15°, 30°, and
45° are extracted and plotted in Fig. 2. To verify the accuracy the new algorithm, split field technique is also used to compute the reflection coefficient of this structure at various incident angles [4]. The results from the constant wavenumber method agree well with data calculated by the split field method.

![Figure 2. Comparison of the reflection coefficients of the dipole FSS calculated using the constant wavenumber method and the split-field method.](image)

**Conclusions**

This paper presents a simple and efficient approach to implement the PBC in the FDTD method. The scattering property of periodic structures is computed in the $k_x$-frequency plane through several rounds of FDTD simulations, each with a constant $k_x$ value. The accuracy of the method is demonstrated through a dipole FSS example.

**References**


