Efficient Analysis of Electromagnetic Scattering Problems Using A Parallel-Multigrid Iterative Multi-Region Algorithm

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Abstract This paper presents the use of multigrid (MG) technique to enhance the solution process of the FDFD method, hence speeding up the computational process of the iterative multi-region (IMR) algorithm. Furthermore, to achieve a high-performance computing, the presented IMR-MG technique becomes more efficient on a parallel platform, through the usage of multiprocessing, where each sub-region is solved on a separate processor. An example of the scattering from 2 two-dimensional objects using the IMR algorithm with multigrid technique in conjunction with parallel processing is presented to show the capabilities of the presented procedure to analyze large scattering problems with reasonable computer resources.

Introduction

Numerical analyses of large-scale electromagnetic problems require long computational time and large computer memory. One of the goals of the ongoing computational electromagnetic research is to develop time and memory efficient algorithms in order to solve real life problems. An iterative multi-region technique (IMR) is recently proposed in [1-2] to efficiently solve such kind of problems, where one computational domain consisting of separable scatterers may be divided into smaller sub-regions and the solution of each sub-region can be performed separately. Then the sub-region solutions are combined iteratively to obtain a solution for the complete domain.

When dealing with large size problems, it is sometimes impossible to fit the number of unknowns in one matrix form and perform a direct matrix inversion solution. Thus different iterative solvers were introduced to solve such problem where they proved their efficient performance regarding memory and time. Multigrid technique (MG) is considered as one of those iterative solvers that can be used to solve this type of problems. It was widely spread starting from early 1970’s by Brandt [3], where it was introduced to perform a fast numerical simulation to the solution of boundary value problems. The multigrid method is an efficient technique generally used for solving smooth partial differential equations (PDEs) [4-6]. Initial interest in the multigrid method was geared towards overcoming the slow convergence rate of the classical iterative methods by updating blocks of grid points.

In this paper the multigrid technique in conjunction with the IMR algorithm is developed and presented to solve large electromagnetic two-dimensional scattering problems, where the solution technique is based on the FDFD method. To achieve additional performance enhancement, the presented IMR-MG algorithm is parallelized, where the sub-regions are solved simultaneously; each on a separate processor. Thus a remarkable CPU time saving can be achieved.

Iterative Multi-Region Algorithm

The IMR algorithm is an iterative approach, used to solve the problem of scattering from large two and three-dimensional electromagnetic objects [1-2]. The idea
of the IMR technique is to divide one computational domain consisting of separable scatterers into smaller sub-regions and solve each sub-region separately. Then the sub-region solutions are combined iteratively to obtain a solution for the complete domain. Hence, the solution of fields in the sub-regions is required a number of times equals to the number of iterations between the sub-regions. This technique effectively reduces the size of the required memory, especially for practical and three-dimensional problems. Furthermore, the CPU time reduction can be achieved if the separation between the sub-regions is relatively large and/or coarser grids are used in some of the sub-regions, which may not be possible if only one domain is used for the solution of the entire problem. The details of the IMR technique are addressed in [1-2] where other special treatments are being used to reduce the required CPU time and memory requirements.

**Multigrid Technique**

For large size problems with the number of unknowns represented in one matrix form, a direct matrix inversion solution would normally be very difficult, if not impossible to achieve on current generation of moderate single CPU computer systems. The multigrid technique, as an iterative solver, can thus be considered as a robust way to provide a solution to such problems. The basic idea of the multigrid technique is to divide the computational domain into a number of levels \( (L) \), going from fine to coarser grid levels. The total number of grid points at each level \( (N_L) \) is taken to be \( N_L = 2^L + 1 \) for a square domain. At each level, different discretization is used which is related to the discretization at the preceding finer grid level by \( \Delta_{\text{finer}}/2 \), for uniform meshing where \( \Delta_{\text{finer}} \) is the discretization used at the finer grid level. In each level a relaxation scheme based on an iterative solver is introduced to smooth out the errors. Gauss-Seidel and Jacobi methods are widely used, for their efficiency and high rate of convergence, as relaxation schemes associated with the MG technique. The only draw back of these two iterative solvers is the requirement of diagonally dominant coefficient matrix of the system of equations describing the problem in order for the solution to converge. This is not the case for most of numerical methods based on differential type solutions used for large electromagnetic frequency domain problems. Thus biconjugate gradients stabilized method (BICGSTAB) is used instead as the relaxation scheme for the constructed MG technique presented in this work, where the solution generated at each level is considered as an initial guess to the BICGSTAB to accelerate the solution convergence rate.

Two basic operations are required in the multigrid technique to go through different levels, a *restriction operator*, and a *prolongation operator*. The restriction operator is used to map the data onto a coarser grid level, while the prolongation operator maps the data from coarser to finer grid level based on a cubic analogue. The initial solution is performed on the coarser grid level at a very low computational cost. Next, prolongation operation takes place in addition to some smoothing steps to interpolate the solution to the finest level. Finally, the desired solution is achieved in less computational time relative to other solution techniques.

**Parallel Computing**

Over the last decade, high-performance computing has been achieved through multiprocessing. Many computations can thus benefit from faster execution on parallel processors. Recently MIT Lincoln Laboratory has developed a noble way for implementing parallel computation using Matlab, version 7. Instead of writing an entire new application of Matlab, multiple Matlab applications can run simultaneously to share the computational load of a single Matlab program, where the multiple applications can communicate through some shared files. Matlab message passing interface (MatlabMPI)
model is designed for easy use on PCs with multiple processors. In this work the IMR-MG technique, using the parallel processing tool in Matlab, is used to compute each sub-region simultaneously on a separate processor. The processors then communicate with each other to generate the final solution for the complete problem. The results presented in this paper are for problems that can be divided into two sub-regions and are computed using dual processor personal computer. Significant time saving can be achieved for larger problems which can be divided into more than two sub-regions when using more than two processors on a single or multiple computers.

**Numerical Results**

A dielectric ellipse of relative permittivity equals to 3.4 with 0.615 $\lambda$ semi-minor axis along the x-axis and 5 $\lambda$ semi-major axis along the y-axis is placed at 0.75 $\lambda$ away from a rectangular conducting plate of length 0.58 $\lambda$ and width 1 $\lambda$, as shown in Fig. 1. This configuration is excited by a TM$_z$ plane wave incident from the negative x-axis. The structure is first solved as a full domain problem using the FDFD numerical technique, where the total near field distribution (Fig. 2), and the echo width (Fig. 3) are computed. Figure 3 shows a comparison between the far field results generated from the full domain problem solution and the IMR-MG technique with 0 and 4 iterations between the sub-regions. Good agreement between the full domain results and the IMR-MG technique after 4 iterations is clearly observed.

Computational time comparison is performed to illustrate the advantage of using the parallel processing in solving this problem using the IMR with 4 iterations. The problem was first solved on a single processor using the BICGSTAB function having incomplete LU (ILU) factorization as a preconditioning to speed up the convergence rate of the BICGSTAB function. The problem was then solved using two processors where each was assigned to solve one of the sub-regions of the entire problem domain. Due to the overhead caused by the communications between the processors; a speed up factor of 30% is achieved for this example.

Despite using the IMR algorithm that tremendously reduces the memory requirements for each sub-region relative to the entire problem domain, the use of a preconditioning requires large memory, which cannot be provided for a large-scale problem. On the other hand the multigrid technique requires no additional memory. Thus the configuration depicted in Fig. 1 is solved using the IMR algorithm on two processors,
with and without the multigrid technique in order to record the computational time difference. A time saving of 34 % was achieved using the multigrid technique for this specific example. Even when the ILU preconditioning was used, which accelerates the rate of convergence of the BICGSTAB function, the multigrid technique proved its efficiency with a time saving of 4 %. The time saving for this case, with the ILU preconditioning, is not significant due to the high rate of convergence of the BICGSTAB function in addition to the overhead in the time required by the multigrid technique. However, one can easily notice from the previous analysis that the parallel IMR-MG technique is an optimal choice in order to save time and memory. For the configuration depicted in Fig. 1, a total computational time saving of 52 % is achieved using the parallel IMR-MG technique relative to that of a full domain solution. All simulations in this investigation were performed on a 64 bit, 3.6 GHz Xeon processor with 2GB RAM addressable by Matlab. Only 4 minutes and 32 seconds are required to obtain the solution for this configuration using the parallel IMR-MG algorithm.

Conclusions
In this paper several enhancements to the IMR algorithm for the solution of large electromagnetic problems are proposed. The introduction of the multi-grid technique together with parallel processing added to the IMR algorithm the advantage of speeding up the computational process; resulting in a robust and efficient technique of low memory consumption and high speed up factor. For the provided example CPU time saving of more than 52 % relative to that of the full domain solution is achieved, in addition to more than 63 % memory savings.

References