Radiation Analysis Approaches for Reflectarray Antennas

Payam Nayeri¹, Atef Z. Elsherbeni¹, and Fan Yang¹,²

¹Electrical Engineering Department
The University of Mississippi
University, MS 38677 USA
E-mail: pnayeri@olemiss.edu, atef@olemiss.edu, fyang@olemiss.edu

²Electronic Engineering Department
Tsinghua University
Beijing, China

Abstract

This paper compares two basic methods for analysis of the radiation performance of reflectarray antennas. Two different approaches—array theory and aperture field—are first described, and numerical results are then presented for various reflectarray configurations. The advantages and limitations for each method are discussed, and the numerical results are compared with each other, showing very good agreement. Comparison with full-wave simulations showed that these approaches are time-efficient methods that can accurately calculate the reflectarray antenna’s pattern shape, main-beam direction, beamwidth, and sidelobe and cross-polarization levels. As such, these methods can be efficient tools for antenna engineers for designing and analyzing reflectarray antennas.

Keywords: Antenna array theory; aperture field; antenna radiation patterns; reflectarray

1. Introduction

Reflectarray antennas imitate conventional parabolic reflectors while having a low profile, a low mass, and a flat surface. They combine the numerous advantages of both printed phased arrays and parabolic reflectors to create a new generation of high-gain antennas [1]. Considering the large number of elements on the reflectarray’s aperture, a full-wave simulation of the antenna is quite challenging. Various approaches have therefore been developed to calculate the radiation characteristics of the reflectarray.

Most reflectarray analysis approaches approximate the elements on the reflectarray’s aperture as identical elements that form an array. Array summation or far-field transformation of currents is then used to calculate the radiation pattern of the reflectarray antenna [1-3]. Accurate characterization of the reflection coefficients of the reflectarray’s elements holds a significant importance in the radiation analysis for these approaches. These coefficients are therefore usually obtained with a full-wave simulation technique, using an infinite-array approach that takes into account the mutual coupling between the elements. Alternatively, the current distributions on the reflectarray elements—obtained from the full-wave simulations under the local-periodicity approximation—could be directly used, and the radiation pattern would then be calculated using far-field transformation of the currents for the total array [4]. The great advantage of these analysis methods is the fast computational time. However, it is implicit that due to the approximations in the analysis, some discrepancies with practical results may be observed. On the other hand, although computationally quite challenging, full-wave analysis methods can provide accurate results. The electrically large size of the reflectarray antenna’s aperture, combined with hundreds of elements with dimensions smaller than a wavelength, demands a significantly high computational time and large resources for full-wave analysis [5, 6]. While in many cases a full-wave simulation will be advantageous at the final stage of a design, fast computational approaches are still necessary tools for the initial design. This is in addition to several stages of optimization that may be required for a reflectarray antenna’s design [7, 8].
The main target of this article is to review the classical approaches for reflectarray antenna analysis, and to provide a comparative study of the accuracy, limitations, and challenges in program development for these methods. Two basic methods for analysis of the reflectarray antenna are studied here. In the first method, the radiation pattern of the reflectarray antenna is calculated using conventional array summation with proper element excitation. In the second method, the radiation pattern of the antenna is calculated using the tangential fields on the reflectarray’s aperture. Numerical results are presented for various reflectarray configurations, using both approaches. It is shown that the computed radiation patterns and antenna directivities obtained by both methods are in close agreement. In addition, the radiation patterns computed with these approaches are compared to full-wave simulations, which show good agreement. Finally, the advantages and limitations of each method are summarized, which will help antenna engineers to select proper methods for their own designs.

2. Classical Approaches for Reflectarray Analysis

2.1 Array-Theory Method

Conventional array theory can be implemented to calculate the far-field radiation pattern of the reflectarray antenna. The radiation pattern of a two-dimensional planar array with \( M \times N \) elements can be calculated as

\[
\hat{E}(\hat{u}) = \sum_{m=1}^{M} \sum_{n=1}^{N} A_{mn}(\theta) \times I(\vec{r}_{mn}),
\]

(1)

where \( A \) is the element-pattern vector function, \( I \) is the element-excitation vector function, and \( \vec{r}_{mn} \) is the position vector of the \( mn \)th element [2]. The geometry of the reflectarray system is given in Figure 1.

To simplify calculations, one usually uses scalar functions in the analysis. For the element-pattern function, \( A \), a scalar approximation considers a cosine \( q \) model for each element with no azimuth dependence, i.e.,

\[
A_{mn}(\theta, \phi) = \cos(E) e^{jk(\vec{r}_{mn} \cdot \hat{u})},
\]

(2)

The element-excitation function, \( I(m, n) \), is determined by the incident field and the element properties. By approximating the feed-horn pattern function using a cosine \( q \) model, and taking into account the Euclidian distance between the feed horn and the element, the illumination of the aperture can be obtained. The element excitation can then be expressed as

\[
I(m, n) = \frac{\cos(E) f(m, n)}{\left| \vec{r}_{mn} - \vec{r}_f \right|} e^{-jk\left| \vec{r}_{mn} - \vec{r}_f \right|} \left| \vec{r}_{mn} \right| e^{j\phi_{mn}}.
\]

(3)

Here, \( \theta_f \) is the spherical angle in the feed’s coordinate system, and \( \vec{r}_f \) is the position vector of the feed. In addition, for each element, this excitation can take into account the receiving mode pattern, i.e., \( |\Gamma| \). This pattern is also modeled by a cosine \( q \) function, based on the local element coordinates, i.e.,

\[
|\Gamma| = \cos(E) \theta_f (m, n).
\]

(4)

The required phase delay of the \( mn \)th element, \( \varphi_{mn} \), is designed to set the main beam in the \( \hat{u}_0 \) direction. The infinite-array approach is often used for analysis of reflectarray elements. This approach analyzes a single reflectarray element in an infinite-array environment, with all surrounding elements identical. As a result, \( \varphi_{mn} \) in Equation (3) does contain the mutual-coupling effects in an infinite-array environment.

With these approximations, the radiation pattern from Equation (1) can be simplified to the scalar form:

\[
E(\theta, \phi) = \sum_{m=1}^{M} \sum_{n=1}^{N} \cos(E) \frac{\cos(E) f(m, n)}{\left| \vec{r}_{mn} - \vec{r}_f \right|} e^{-jk\left| \vec{r}_{mn} - \vec{r}_f \right|} \left| \vec{r}_{mn} \right| e^{j\phi_{mn}}.
\]

(5)

The radiation-pattern calculation method described here uses a conventional array-summation technique. In general, the array-theory formulation will yield good main beamwidth, beam direction, and general pattern shape. However, since the polarizations of the feed horn and elements are not accounted for in the simplified cosine \( q \) model, the cross-polarization of the reflectarray antenna is not calculated in the above procedure.

In summary, the advantages and disadvantages of this approach are tabulated here:

Advantages

- Simplicity of the formulation and program development
- Fast computational time

Limitations

- Cross-polarization is not modeled

2.2 Aperture-Field Method

In this method, the radiation pattern of the reflectarray is calculated from the aperture fields using the equivalence principle. First, the incident fields on the aperture’s surface are obtained while considering the polarization of the feed horn. The radiation pattern of an ideal feed horn [9] with a fixed phase center is given by

\[ E_x^f(\theta, \phi) = A_0 \left[ \hat{\phi}_c(\theta) \cos \phi - \hat{\phi}_c(\theta) \sin \phi \right] e^{-jkr} \]

for \( x \) polarization,

\[ E_y^f(\theta, \phi) = A_0 \left[ \hat{\phi}_c(\theta) \sin \phi + \hat{\phi}_c(\theta) \cos \phi \right] e^{-jkr} \]

for \( y \) polarization,

where \( A_0 \) is a complex constant, and \( C_E \) and \( C_H \) are the E- and H-plane radiation patterns of the horn antenna, respectively. \( C_E \) and \( C_H \) are typically modeled as \( \cos^q(\theta) \) functions, where the value of \( q \) is determined from the measured data of the horn antenna. Note that in Equation (6), the cross-polarization of the feed horn is not taken into account, which may be considered by expanding the feed model. It should be noted that the radiation patterns in Equation (6) are given in the feed’s coordinate system. From these equations, the fields on the reflectarray’s aperture are obtained using matrix transformations from the feed to the array coordinates, as described in [10]. Alternatively, the fields incident on the reflectarray’s aperture can be obtained directly from the horn-antenna simulation, or from measurements. This approach is particularly more suitable when a highly accurate estimation of the cross-polarization level of the reflectarray antenna is required.

From the incident fields on the reflectarray’s aperture, the reflected fields for every element in the array are obtained using the generalized scattering matrix relating the Cartesian components of incident and reflected fields in the periodic cell, i.e.,

\[
\begin{bmatrix}
E_x^{ref}(m,n) \\
E_y^{ref}(m,n)
\end{bmatrix} = 
\begin{bmatrix}
\Gamma_{xx} & \Gamma_{xy} \\
\Gamma_{yx} & \Gamma_{yy}
\end{bmatrix} 
\begin{bmatrix}
E_x^{inc}(m,n) \\
E_y^{inc}(m,n)
\end{bmatrix}
\]

(7)

The phase shift produced for each element depends on the polarization of the feed, and also takes into account the cross-polarized reflected fields. With the aperture-reflected fields specified on the reflectarray’s surface, the far-field radiation pattern can be calculated using the vector potentials as described in [11]. For the element analysis, the infinite-array approximation is used to obtain the scattering matrix in Equation (7). As discussed, since the equations in this approach are in a vector form, the cross-polarized reflected field of each element is also taken into account.

One important approximation in this approach is that the tangential currents are assumed to be constant within each element, so the integration over the aperture surface is approximated by a double summation. In general, this double summation must be extended to the entire plane of the reflectarray’s aperture, but it is always limited to the reflectarray’s surface with an assumption that the tangential currents are zero outside the reflectarray. It is worthwhile to mention that to simplify the calculations, usually only the electric-field components are used in the analysis. This corresponds to the second principle of equivalence, i.e., assuming the antenna aperture is immersed in a perfect electric conductor.

The advantages and disadvantages of this approach are tabulated here:

Advantages

- Accurate modeling of feed and element polarization

Limitations

- Complicated formulation and program development
- Increased computational time

2.3 Computational Speedup Using Spectral Transforms

In the previous sections, it was shown that both approaches require evaluation of a double summation for far-field calculations. This double summation can be replaced by a two-dimensional inverse discrete Fourier transform, defined as

\[ f(p,q) = \frac{1}{N_x N_y} \sum_{m=0}^{N_x-1} \sum_{n=0}^{N_y-1} F(m,n) e^{j2\pi px/N_x} e^{j2\pi qy/N_y} \]

(8)

Here, the spectral functions will be obtained in a discrete number of angular coordinates. These points in the \((u,v)\) plane are defined by the Fourier transform as
\[-\frac{2\pi}{N_xd_xk_0} p , \ p = 0,1,2,\ldots N_x - 1; \]
\[-\frac{2\pi}{N_yd_yk_0} q , \ q = 0,1,2,\ldots N_y - 1. \]

The main advantage of using spectral functions in the calculations is a significant reduction of computational time, which is simply achieved by replacing the double summations with Fourier transforms. It should be noted that the definition of the propagating-wave direction determines whether the discrete Fourier transform (DFT) or inverse discrete Fourier transform (IDFT) is to be used for the transforms. In the formulation presented here, we followed the definition in [2], where the inverse discrete Fourier transform is used to replace the double summation in the spectral functions.

3. Comparison of the Analysis Approaches

The techniques presented in Section 2 were applied to Ka-band reflectarrays for comparison of these methods. We considered a Ka-band reflectarray with a circular aperture and a diameter of \(17\lambda\) at the design frequency. The phasing elements used in this study were variable-size square patches, with a unit-cell periodicity of \(\lambda/2\) at the design frequency of 32 GHz. They were fabricated on a 20 mil Rogers 5880 substrate. The reflection-phase response (S curve) of the phasing elements obtained using the infinite-array approach was generated using Ansoft Designer [12], and is given in Figure 2. It could be seen that the reflection characteristics of the phasing elements were angle dependent. However, for this design, normal incidence could present good approximations for oblique-incidence angles up to 30°. The reflectarrays here were thus designed based on the simulated reflection coefficients obtained with normal incidence.

3.1 Comparison of Calculated Radiation Pattern

Two different reflectarray systems were studied. In the first case, the reflectarray phasing elements were designed to generate a beam in the broadside direction. The \(x\)-polarized prime-focus feed horn was positioned with an \(F/D\) ratio of 0.735. For the horn model used in this study, the power, \(q\), of the feed radiation pattern was 6.5 at 32 GHz. As discussed previously, in the array-theory calculations, the polarization of the feed horn was not modeled. The principal plane (P.P.1 and P.P.2) radiation patterns of the reflectarray antenna [13] calculated by both methods are given in Figure 3 at 32 GHz. It should be noted that with this design, the cross-polarized pattern obtained using the aperture-field formulation was almost zero in the principal planes. The maximum cross-polarization level for this system was \(-36.1\) dB, which occurred in the 45° planes.
For the second case, we considered designing a reflectarray with an off-broadside beam. The phasing elements were designed to generate a beam in the direction of $(\theta, \phi) = (25^\circ, 0^\circ)$. The offset feed horn position was $x_{\text{feed}} = -45.9$ mm, $y_{\text{feed}} = 0$ mm, $z_{\text{feed}} = 98.4$ mm, based on the coordinate system in Figure 1. The feed horn was left-hand circularly polarized (LHCP), and the power, $q$, of the feed radiation pattern was 6.5 at 32 GHz. The radiation patterns in the principal planes at 32 GHz are given in Figure 4. For this system, the cross-polarized radiation pattern was also observed in both principal planes, where the maximum cross-polarization level was $-30.0$ dB.

From the results presented in Figs. 3 and 4, it could be seen that the calculated radiation patterns obtained by both methods were in close agreement with each other. In particular, the main-beam directions, beamwidths, and general pattern shapes were almost identical. However, a slight difference was observed in the sidelobe regions.

### 3.2 Comparison of Calculated Directivity as a Function of Frequency

The antenna’s directivity is a suitable measure for comparing the calculated radiation performance of these methods. In order to accurately model the reflectarray directivity as a function of frequency, the frequency behavior of the feedhorn pattern and of the elements’ reflection characteristics were implemented into this calculation routine. For the phasing elements, the frequency behavior of the reflection phase was obtained across the band from full-wave simulations. For the horn model used in this study, the power, $q$, of the feed radiation pattern varied linearly, from 5 at 30 GHz to 8.3 at 34 GHz, according to the measured data [14]. For the two reflectarray systems studied in the previous section, the directivity as a function of frequency is given in Figure 5. It could be seen that the computed directivity as a function of frequency obtained by both methods also showed a close agreement. At the center frequency of 32 GHz, the difference in computed directivity was less than 0.1 dB for both designs. It should be noted here that for the two designs studied in this section, the off-broadside system showed a lower directivity and a slightly larger bandwidth.

### 4. Classical Approaches Compared to Full-Wave Simulations

In the previous sections, two basic techniques were presented for reflectarray radiation analysis. Different reflectarray systems were studied, and the numerical results showed that the radiation patterns computed by both methods were in close agreement with each other. However, as discussed in Section 2, several approximations were considered in these approaches. In addition, the reflectarray phasing elements were analyzed with an infinite-array approach, which was also an approximation.
As a result of these approximations in the analysis, it was expected that the radiation pattern obtained by these approaches might show some discrepancy with practical results. The aim of the study here was to analyze the accuracy of the reflectarray radiation pattern computed using these approaches by comparing them with full-wave simulations. For this comparative study, full-wave simulations were more advantageous than measured results, since measured results are susceptible to both fabrication and measurement errors. Here, we considered a Ka-band reflectarray with a circular aperture and a diameter of 14.5\,\lambda at the design frequency of 32 GHz. The feed was positioned at \( x_{\text{feed}} = -45.90 \,\text{mm}, \ y_{\text{feed}} = 0 \,\text{mm}, \ z_{\text{feed}} = 98.44 \,\text{mm}, \) based on the coordinate system in Figure 1.

The phases of the elements are designed to generate a beam in the direction of \((\theta, \phi) = (25^\circ, 0^\circ)\). For the reflectarray’s phasing elements, variable-size square patches were selected from the S-curve data in Figure 2. The 609-element reflectarray antenna was modeled using the commercial electromagnetic software FEKO [15]. For the excitation of the reflectarray, a point-source feed model with a \( \cos^{6.5}\,\theta \) radiation pattern was used. The advantage of using a point source rather than a feed horn here was that a point-source model does not have a blockage aperture. This makes it more suitable for comparison purposes, since blockage is typically not modeled in the classical methods. For this design, 568,435 unknown basis functions had to be calculated for the FEKO Method of Moments (MoM) solution. Considering the large number of unknowns, the Multilevel Fast Multipole Method (MLFMM) solver in FEKO was selected for this simulation. The geometry of the reflectarray antenna modeled in FEKO and the simulated three-dimensional radiation pattern are shown in Figure 6. The full-wave simulation here took into account all approximations in reflectarray element design and mutual coupling, in addition to edge-diffraction effects. Therefore, comparing these simulation results with the results obtained using classical approaches could provide a good measure in terms of the accuracy of these approaches.

The principal-plane (P.P.1 and P.P.2) radiation patterns of the reflectarray antenna, calculated using the aperture-field method and the full-wave simulation, are given in Figure 7. Comparison of the results given here showed that the analysis approaches presented in this paper can accurately calculate the general pattern shape, the main-beam direction, the beam-width, and the sidelobe and cross-polarization levels in the main-beam area. However, outside the main-beam area, some discrepancies were observed among these results. These were primarily due to element-design approximations, mutual coupling, and edge-diffraction effects that were not taken into account in the analysis approaches discussed in Section 2. In particular, comparisons between the ideal phase shift and the phase shift obtained by the reflectarray’s elements (Figure 8) indicated that while the reflectarray’s aperture did indeed generate a phase shift that creates the collimated beam, this difference in phase shift was the primary reason for the discrepancies in the radiation patterns. It should be noted that the calculated co-polarized radiation pattern of the reflectarray antenna obtained using the array-theory approach was similar to the aperture-field results, and is not shown in this section for the sake of brevity.

While it is implicit that a full-wave simulation will provide accurate results, the main disadvantages of full-wave simulations are the high computational time and resources required. In total, the full-wave simulation here required 29.56 GB of memory with a CPU time of 26.97 hours on an eight-core 2.66 GHz Intel Xeon E5430 computer. In comparison, the CPU times for the array-theory and aperture-field approaches in the two methods were completely different, but the limitations for each method were described, and the numerical results with the results obtained using classical approaches could provide a good measure in terms of the accuracy of these approaches.
in the radiation patterns. It should be noted that the calculated
phase shift was the primary reason for the discrepancies.
A phase shift that creates the collimated beam, this differ-
ence in phase shift was the primary reason for the discrepan-
cies. Considered that while the reflectarray’s aperture did indeed gen-
erate a phase shift, this difference in phase shift was the primary reason for the discrepan-
cies. In addition to the computational time, a major limitation in full-
wave simulations is the available memory of the computer.

5. Conclusion

Two basic methods for radiation analysis of the reflectar-
ray antenna were studied in this work. The advantages and
limitations for each method were described, and the numerical
results were compared with each other. While the analysis
approaches in the two methods were completely different,
the results presented here show that for a fast analysis of the
reflectarray antenna’s radiation characteristics, the array-theory
approach is a suitable method. This is because the co-polarized
radiation pattern and the directivity obtained were in close
agreement with the aperture-field approach. For a more-accurate
modeling of the reflectarray antenna where analysis of the cross-
polarization is also necessary, one could use the aperture-field
approach at the expense of increasing the computational time
and programming complexity. While several approximations
were considered in these approaches, comparison with full-
wave simulations showed that by using these methods one
can accurately calculate the general pattern shape, the main-
beam direction, the beamwidth, and the sidelobe and cross-
polarization levels in the main-beam area. This makes them
efficient analysis tools for antenna engineers for evaluating the
performance of their own reflectarray designs. This study also
revealed that in general, the classical approaches have limited
accuracy. When accurate radiation-pattern computations in
the entire three-dimensional space are required, full-wave
simulation of the reflectarray system is necessary.
6. Acknowledgment

The authors acknowledge EM Software & Systems (USA) Inc. for providing us with a full evaluation version of FEKO v.6.1. This work was supported in part by NASA EPSCoR program under contract number NNX09AP18A.

7. References


Ideas for Antenna Designer’s Notebook

Ideas are needed for future issues of the Antenna Designer’s Notebook. Please send your suggestions to Tom Milligan and they will be considered for publication as quickly as possible. Topics can include antenna design tips, equations, nomographs, or shortcuts, as well as ideas to improve or facilitate measurements.