Iterative multiregion technique for large-scale electromagnetic scattering problems: Two-dimensional case

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[1] One of the methods to solve large electromagnetic problems is to divide the computational domain into smaller subregions and solve each subregion separately. Then the subregion solutions are combined to obtain a solution for the complete domain. In this paper, an iterative approach using the finite difference frequency domain method is presented to solve the problems that can be divided into separated subregions.


1. Introduction

[2] Numeric analysis of large-scale electromagnetic problems requires large computer memory and long computation time. One of the approaches to overcome this problem is to divide the computational domain into smaller subregions and then to combine the subregion solutions after introducing the effect of interactions between these subregions. Another approach is to use more than one computing platform with parallel processing features that allows for the accessing more memory and CPU resources, but at the expense of restructuring the solution algorithm and the development of communication protocols between the different processors. Among the available techniques that tackle one class of these large-scale problems is the domain decomposition method (DDM) [Després, 1992a, 1992b; Stupfel, 1996; Yin and Hong, 1999; Zhu et al., 1997; Gander, 2001; Lee and Chupongstimun, 1994; Spring and Cangellaris, 1995], which in general requires either common boundaries or overlapping regions between the subdomains. In this technique a large problem is decomposed into coupled subdomains each of which can be analyzed separately and validity of the solution for the complete domain is ensured by satisfying certain boundary conditions in between the subdomains. Thus the most appropriate solution algorithm can be used in each subdomain to achieve better performance. The drawback of this technique is that it assumes connected subdomains in order to apply the appropriate boundary conditions that are needed to provide the interaction between the subdomains. Because of this restriction, this technique has been in use, in general, for electromagnetic problems with closed or semiclosed boundaries. In order to economically provide efficient solution to large-scale electromagnetic problems, especially those that involve open boundaries such as the scattering from multiple objects, decomposing the computation domain into separate subregions would be preferable. It is then necessary to develop accurate procedures to support the interaction between the unconnected subregions. Some hybrid techniques based on combinations of method of moments (MOM), finite element (FE), finite difference time domain (FDTD), and physical optics (PO) have been used to solve a class of these problems, in which part of the problem is usually large compared to other parts [Thiele, 1992; Carr and Volakis, 2001; Monorchio et al., 2004]. Despite that most of the approaches used hybrid techniques to solve the problem of scattering from unconnected objects; domain decomposition using FDTD method [Xu and Hong, 2004] and using fast multipole method [Yin et al., 2000], were used to solve the problem of scattering from multicylinders. Although these two papers do not show the effect of the interaction between the cylinders; as the examples presented indicate that the separation distance between the objects is more than 5 times the largest dimension of one object, in the first paper, and almost 300 times in the second paper. Such separation distances does not require any interaction between the objects, simple superposition of individual scattering effect would be sufficient.

[3] In this paper, we present a new technique based on the finite difference frequency domain (FDFD) method and an iterative procedure between the subregions to
calculate the scattering from multiple objects similar to that described by Elsherbeni et al. [1993]. The problem is decomposed into separated subregions, each subregion containing a scatterer or a group of scatterers. In each subregion, the scattered electromagnetic near fields are calculated on the basis of the incidence of a time-harmonic wave, using the FDFD method. Then fictitious electric and magnetic currents on imaginary surfaces surrounding the objects in these subregions are calculated, using the equivalence principle. Radiated fields by these currents are then considered as incident fields on the opposing subregions. The same procedure of calculating the subregion field components, the fictitious currents and the radiated fields on the opposing regions is repeated iteratively until a convergence criterion is achieved. The iterative procedure developed here is similar to the procedure denoted as iterative field bouncing (IFB) method and described briefly by Carr and Volakis [2001], Reuster and Thiele [1995], and Obelleiro-Basteiro et al. [1995], where the last two references used the iterative method to compute the scattered field from a large perfectly conducting cavity using integral equations and physical optics, respectively.

[4] The procedure presented in this paper, referred to as iterative multiregion (IMR) technique, requires solution of fields in the subregions a number of times instead of one solution of the complete computational domain. This technique effectively reduces the size of the required memory, especially for practical and three-dimensional problems. Furthermore, the CPU time reduction can be achieved if the separation between the subregions is large and/or coarser grids are used in some of the subregions, which may not be possible to use if only one domain is used for the solution of the problem. In this paper the application of this technique will be applied on two-dimensional scatterers. The verification of the proposed technique and the validity of the generated numerical data is performed by comparison with the corresponding near- and far-field data of the scattering from two cylinders on the basis of a rigorous boundary value solution approach for the scattering from multiple cylinders [Elsherbeni, 1994]. The use of the FDFD provides the flexibility in defining composite (in shape and type) structures in each subregion. It also provides a much more stable solution relative to other available methods and a more convenient procedure for performing the interaction between the subregions on the basis of well-known theorems. The use of the FDTD technique was intentionally avoided in this work as the interaction process between subregions would involve complex bookkeeping for the source and its effect in the spatial and time domains from one subregion to the other. The FDFD solution provided in this paper is following the analysis of Wang [1995]. However, in the present formulation, only the $E_z$ component is used in the resulting matrix equation instead of using $E_z$, $H_x$, $H_y$ as presented by Wang [1995]. This allows for additional memory reduction and hence the possibility of analyzing much larger problems.

2. Two-Dimensional FDFD Formulation

[5] Starting from Maxwell’s equations for the total electric and magnetic fields

$$\nabla \times \mathbf{E}^\text{tot} = -j\omega \mu \mathbf{H}^\text{tot}, \quad \nabla \times \mathbf{H}^\text{tot} = +j\omega \varepsilon \mathbf{E}^\text{tot}$$

and then by separating the total field into incident and scattered field components, we obtain

$$\nabla \times \left( \mathbf{E}^i + \mathbf{E}^s \right) = -j\omega \mu \left( \mathbf{H}^i + \mathbf{H}^s \right),$$

$$\nabla \times \left( \mathbf{H}^i + \mathbf{H}^s \right) = +j\omega \varepsilon \left( \mathbf{E}^i + \mathbf{E}^s \right).$$

The superscripts $i$ and $s$ are used to denote the incident and scattered fields, where the incident field is the field that would exist in the computational domain with no scatterers. If the computational domain is free space then the incident field satisfies Maxwell’s equations, such that

$$\nabla \times \mathbf{E}^i = -j\omega \mu_0 \mathbf{H}^i, \quad \nabla \times \mathbf{H}^i = +j\omega \varepsilon_0 \mathbf{E}^i.$$

Substitution of equations (3) into (2) yields

$$\mathbf{H}^i + \frac{1}{j\omega \mu} \nabla \times \mathbf{E}^i = \left( \frac{\mu_0}{\mu} - 1 \right) \mathbf{H}^i$$

$$\mathbf{E}^s - \frac{1}{j\omega \varepsilon} \nabla \times \mathbf{H}^s = \left( \frac{\varepsilon_0}{\varepsilon} - 1 \right) \mathbf{E}^s.$$

In this paper, the FDFD formulation for the two-dimensional TMz case is given and the presented method is applied to problems that belong to this case, without loss of generality. The incident TMz plane wave field components can be given as

$$E^i_x(x,y) = E_0 e^{jk_0 \left( x \cos \phi + y \sin \phi \right)}$$

$$H^i_y(x,y) = -\sin \phi \sqrt{\frac{\varepsilon_0}{\mu_0}} E_0 e^{jk_0 \left( x \cos \phi + y \sin \phi \right)}$$

$$H^i_z(x,y) = \cos \phi \sqrt{\frac{\varepsilon_0}{\mu_0}} E_0 e^{jk_0 \left( x \cos \phi + y \sin \phi \right)},$$

where $E_0$ is the magnitude of the incident electric field, $k_0$ is the wave number, $\varepsilon_0$, and $\mu_0$ are the permittivity and permeability of free space. The incident angle with respect to the $x$ axis of the global coordinates system is $\phi^i$. Having defined the incident fields, equation (5) can
be written for the $z$ component of the scattered electric field in the form
\[
E^s_z = -\frac{1}{j\omega\varepsilon_{xz}} \frac{\partial H^s_y}{\partial x} + \frac{1}{j\omega\varepsilon_{zy}} \frac{\partial H^s_x}{\partial y} = \left(\frac{\varepsilon_0}{\varepsilon_{zi}} - 1\right) E^s_z. \tag{7}
\]

Using equation (4) and assuming that $E^s_x = 0$ and $E^s_y = 0$ (for the TM$_z$ case); $H_x$ and $H_y$ can be expressed in terms of the $z$ component of the scattered electric field as
\[
\begin{align*}
H^s_x &= -\frac{1}{j\omega\mu_{xi}} \frac{\partial E^s_z}{\partial x} + \left(\frac{\mu_0}{\mu_{xi}} - 1\right) H^s_x, \\
H^s_y &= -\frac{1}{j\omega\mu_{yi}} \frac{\partial E^s_z}{\partial y} + \left(\frac{\mu_0}{\mu_{yi}} - 1\right) H^s_y. \tag{8}
\end{align*}
\]

In equations (7) and (8) the permittivity and permeability parameters are indexed in such a way that these equations will be used for the perfectly matched layer (PML), that will be used to truncate the computational domain, and the non-PML regions as well. These parameters defined in different ways depending on whether the node at which the fields are to be evaluated is inside the PML region or outside, as follows.

[6] In the PML region,
\[
\begin{align*}
\varepsilon_{zi} &= \varepsilon_0 - j\frac{\sigma^e_{zi}}{\omega}, & \mu_{xi} &= \mu_0 - j\frac{\sigma^m_{xi}}{\omega}, \\
\varepsilon_{zy} &= \varepsilon_0 - j\frac{\sigma^e_{zy}}{\omega}, & \mu_{yi} &= \mu_0 - j\frac{\sigma^m_{yi}}{\omega}, \\
\varepsilon_i &= \varepsilon_0, & \mu_i &= \mu_0.
\end{align*}
\]

[7] Outside the PML region,
\[
\begin{align*}
\varepsilon_{zi} &= \varepsilon_{zi}, & \mu_{zi} &= \mu_{zi}, \\
\varepsilon_{zy} &= \varepsilon_{zy}, & \mu_{zy} &= \mu_{zy}.
\end{align*}
\]

where $\sigma^e_{zi}, \sigma^e_{zy}, \sigma^m_{xi}$ and $\sigma^m_{yi}$ are the PML electric and magnetic conductivity distributions and $\varepsilon_{zi}, \mu_{zi}$ and $\mu_{zy}$ are the anisotropic material parameters within the non-PML region of the computational space. Equations (7) and (8) can be reduced to one equation, which can be written in terms of the $z$ component of the scattered electric field as
\[
E^s_z = \frac{1}{j\omega\varepsilon_{zi}} \frac{\partial E^s_z}{\partial x} + \frac{1}{j\omega\varepsilon_{zy}} \frac{\partial E^s_z}{\partial y} = \left(\frac{\varepsilon_0}{\varepsilon_{zi}} - 1\right) E^s_z - \frac{1}{j\omega\mu_{xi}} \frac{\partial}{\partial y} \left(\frac{\mu_0}{\mu_{xi}} - 1\right) H^s_x - \frac{1}{j\omega\mu_{yi}} \frac{\partial}{\partial x} \left(\frac{\mu_0}{\mu_{yi}} - 1\right) H^s_y. \tag{9}
\]

The central finite difference approximation of equation (9), takes the following general form
\[
a_{(ij)}E_{(i,j)} + b_{(ij)}E_{(i,j-1)} + c_{(ij)}E_{(i,j+1)} + d_{(ij)}E_{(i+1,j)} + e_{(ij)}E_{(i+1,j-1)} = f_{(ij)}, \tag{10}
\]

where the subscript “$z$” and the superscript “$s$” are omitted for simplifying the presentation of the equation, and the coefficients $a, b, c, d,$ and $e$ are defined as
\[
an = \frac{1}{(\Delta x)^2 \varepsilon_{zi} \mu_{zi} \omega}; \quad bn = \frac{1}{(\Delta y)^2 \varepsilon_{zi} \mu_{zi} \omega}; \quad cn = \frac{1}{\mu_i \omega}; \quad dn = \frac{1}{\varepsilon_{zi} \mu_{zi} \omega}; \quad en = \frac{1}{\mu_i \omega},
\]

while the permittivities at one half cell are given by
\[
\begin{align*}
\varepsilon_{zi} &= \frac{1}{2} \left(\varepsilon_{zi} + \varepsilon_{zi}\right), \quad \varepsilon_{zi} = \frac{1}{2} \left(\varepsilon_{zi} + \varepsilon_{zi}\right) \quad \varepsilon_{zi} = \frac{1}{2} \left(\varepsilon_{zi} + \varepsilon_{zi}\right) \quad \varepsilon_{zi} = \frac{1}{2} \left(\varepsilon_{zi} + \varepsilon_{zi}\right),
\end{align*}
\]

[8] The electric and magnetic field components are defined on the two-dimensional grid as shown in Figure 1. The magnetic field components are off grid half a cell from the nodes representing the electric field.

[9] A linear set of equations can be constructed using equation (10) based on the node scheme given in Figure 1. These equations can be arranged in a matrix form as $\mathbf{A}\mathbf{E} = \mathbf{Y}$, where $\mathbf{A}$ is a highly sparse coefficents matrix of order $N$, where $N$ is the number of nodes in the computational domain, $\mathbf{E}$ is the unknown vector, in which each element represents one of the $E^s_z$ scattered electric field component in the computation grid, and $\mathbf{Y}$ is the excitation vector representing the right hand side of equation (10) and is a function of incident field components, $E^i_z, H^i_x,$ and $H^i_y$. The solution of this matrix equation for the vector $\mathbf{E}$ yields the $E^s_z$ field components in the computational grid. The sparsity pattern of $\mathbf{A}$ can be observed as shown in Figure 2 for an example of a small computational domain of $(7 \times 7)$ nodes in $x$ and $y$ direction ($N = 49$).

[10] A sparse matrix solver can be used to efficiently solve this kind of matrix equations. In our analysis the sparse matrix definition in Matlab is being used along with the command (sparse) to reduce the matrix storage by keeping in memory only the nonzero elements of matrix $\mathbf{A}$.

3. Iterative Procedure Between Multiple Regions

[11] The iterative technique developed here is based on dividing the original electromagnetic scattering problem
of a large domain into smaller problems in separated subregions and then an interaction between the small problems is to be taken into consideration. Dividing the original problem into smaller problems provides the benefit of minimizing the domain size and thus saving a huge memory in addition to the saving in the computational time, especially with large separation between some of the smaller domains. Therefore instead of dealing with one large domain one would be dealing with multiple smaller regions.

[12] Consider a problem as shown in Figure 3. The first step is to divide the original problem computational domain into subregions, in this case region 1 and region 2. The scattered electromagnetic fields due to an incident wave are calculated separately in each subregion by the

**Figure 1.** Node scheme for a TM subdomain two-dimensional scattering problem. See color version of this figure in the HTML.

**Figure 2.** Sparsity pattern of matrix A for a (7 × 7) computational domain. See color version of this figure in the HTML.

**Figure 3.** Original domain of a large-scattering problem showing possible decomposition to two subregions.

**Figure 4.** Scheme for the iterative procedure applied by converting the electric and magnetic currents to field components generated on the other region.
FDFD method. Then fictitious electric and magnetic currents are calculated over imaginary surfaces surrounding the objects in each subregion on the basis of the equivalence principle. The electromagnetic fields radiated by these currents are calculated at the other subregions grid nodes. Then these fields are considered as the new excitation for that region and the cycle of calculation of scattered fields, fictitious currents and radiated fields are repeated as a new iteration. The iteration process between subregions continues until a convergence criterion is achieved. The sum of all calculated scattered fields through iterations gives the total scattered field, which is equivalent to the scattered field calculated from the solution of the original problem. This iterative procedure is illustrated in Figure 4.

4. Numerical Results

4.1. Verification Configuration

[13] Figure 5 shows the geometry of a two-dimensional TM$_z$ problem of two, one conducting and one dielectric, cylinders of circular cross section separated by a distance 0.4 $\lambda$, where $\lambda$ is the free space wavelength that is assumed to be equal to 1 m in this analysis. The conductive cylinder has a radius of 0.1 $\lambda$ while the dielectric cylinder radius is 0.5 $\lambda$, the latter has relative permeability $\mu_r = 1$ and relative permittivity $\varepsilon_r = 5$. The incident electromagnetic plane wave field components on the cylinders are given by equation (6). The problem in Figure 5 is used to verify the validity of the IMR technique described in this paper, since the exact solution of this problem can be obtained by the boundary value solution (BVS) [Elsherbeni, 1994].

[14] Figure 6 shows the bistatic echo width calculated from the scattered $E_z$ field component for the problem illustrated in Figure 5, where the two cylinders are excited by a TM$_z$ plane wave having $\phi' = 90^\circ$ from the positive x axis. The data in Figure 6 are computed using three different methods; first, the boundary value solution (BVS) [Elsherbeni, 1994], second the FDFD solution for the full domain and third the IMR procedure using FDFD in each subregion. It can be seen that, the IMR technique results converge to the full domain.

![Figure 5](image.png)

**Figure 5.** Geometry of one conducting and one dielectric cylinder. See color version of this figure in the HTML.

![Figure 6](image.png)

**Figure 6.** Comparison of the far field using the boundary value solution (BVS) and finite difference frequency domain (FDFD) for the full domain and FDFD for subregions after two, four, and six iterations for the problem defined in Figure 5. See color version of this figure in the HTML.
Figure 7. Comparison of the near-field distributions using FDFD for (a) full domain and (b) subregions. See color version of this figure in the HTML.

Figure 8. Geometry of the test configuration. See color version of this figure in the HTML.
solutions as the number of iterations is increased. Because of the flexibility in discretizing the subregions, different cell sizes are used for the two subregions; the cell size used in the left region is 2 cm on a side, while it is 1 cm on a side in the right subregion. The cell size used in the full domain solution is 1 cm on a side. The number of cells in the full domain is 27800 while the total number of cells in the two subdomains is 11402. Therefore 59% memory reduction in the storage requirements for this problem configuration is achieved by using the IMR technique. The computation time of the full domain solution is 1.30 min while total solution time of the two subregions is 2.37 min after four iterations using Matlab version 13 on a 1.9 GHz P4 personal computer.

[15] Figure 7 shows a comparison between the near-field distributions generated using the FDFD code for the full domain and the FDFD code for the two separate subregions using the IMR technique. The figures generated for the subregions are without the absorbing bound-

Figure 9. Comparison of the total near field generated along the S-S plane cut shown in Figure 8. See color version of this figure in the HTML.

Figure 10. Comparison of the near-field distributions using FDFD for (a) full domain and (b) subregions. See color version of this figure in the HTML.
4.2. Test Configurations

[16] A nonmagnetic rectangular dielectric cylinder having relative permittivity equal to 5 is embedded in a U shaped conductor plate. A magnetic circular cylinder of relative permittivity equal to 2 and radius equal to 0.25\(\lambda\) is placed at 0.75\(\lambda\) away from the first object, as shown in Figure 8. This configuration is excited by a TM\(_z\) plane wave with \(\phi^i = 90^0\). Shown in Figure 9 is a 2-D plot representing a cut along the center of the domain for the total \(E_z\) field component, that is calculated from the full domain problem and compared to that generated from the results of the subregions. The cell size used for the full domain is 1 cm on a side while those for the subregions are 1 and 1.25 cm for the first and second regions, respectively. Thus the total size for the full domain problem is 23,496 cells, while for the two regions is 18,385 cells, which is 22% less than that of the full domain. Good agreement is achieved with sufficient reduction in memory size.

[17] Figure 10 shows a comparison between the near-field distributions generated using the FDFD code for the full domain and the FDFD code for two separate subregions using the IMR technique. The figures generated for the subregions are without the absorbing boundary (PML) and they are to be compared with the parts that are surrounded by the dotted lines in the full domain plot. Figure 11 shows a comparison between the echo width \(\sigma\) based on the computed far field using the FDFD code for the full domain and the FDFD using the subregions. Good agreement is achieved at all observation angles after 6, 18, and 26 iterations. Figure 11 shows the stability of the convergence of the solution after a large number of iterations, where a mathematical proof of the convergence has not been developed, as it is difficult to develop such a proof for integral equation-based methods.

[18] In order to show a more general overview on the interaction between multiple regions, the scattering from three cylinders is presented in terms of three separate subregions, and the results are compared to those generated using the boundary value solution method for the three cylinders in one computational domain. As previously mentioned the idea behind the IMR technique is to excite each subregion with the calculated fields, generated because of the computed electric and magnetic currents from the other subregions. As an example, for the three objects, the fields exciting subregion 1 are a result of the superposition of the fields generated on subregion 1 using the currents calculated from subre-

![Figure 11](image)

**Figure 11.** Comparisons of the far field using FDFD for the full domain and FDFD with iterative multiregion (IMR) for subregions after 6, 18, and 26 iterations for the problem defined in Figure 8. See color version of this figure in the HTML.

![Figure 12](image)

**Figure 12.** Geometry of three identical dielectric cylinders oriented along the x axis. See color version of this figure in the HTML.
regions 2 and 3. This process is being repeated for the other subregions for several iterations, until the convergence criterion is being achieved. Figure 12 shows the configuration of the three dielectric cylinders having each relative permeability $\mu_r = 1$ and relative permittivity $\varepsilon_r = 3$, and radius equal to 0.5λ. The cylinders are oriented along the x axis with a 0.5λ separation between each one and are excited by a TM$_z$ plane wave with $\phi_i = 90^\circ$. The generated bistatic echo width for this configuration is shown in Figure 13, where the IMR results using the FDFD method converge to the computed results generated using the boundary value solution method after four iterations. Good agreement between the two methods is recognized at all observation angles.

[19] In all the examples presented here, the number of iterations was determined on the basis of the difference between the maximum values of the computed $E_z$ in two successive iterations. A 1% difference was used to stop the iteration process. The number of air buffer and PML layers used in all the presented problems are 10 and 8, respectively.

5. Conclusions

[20] In this paper an iterative multiregion technique is proposed to solve large-scale electromagnetic problems that can be decomposed into separate subregions using the FDFD method. This procedure starts by dividing the original computational domain into separate subregions where the solution is easily performed by the FDFD method followed by an iterative interaction process between the subregions. The new approach proposed here is found to be efficient in producing accurate results for the original problem with moderate saving in computer memory usage. The two-dimensional problem presented in this paper is a test bed for the more practical three-dimensional configurations, for which significant memory saving is expected. The extension of this technique to 3-D problems is currently under investigation with very promising preliminary results.

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